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"The Nepali Mathematical Sciences Report" will publish the original research papers, short notes and survey articles in different branches of Mathematics and Mathematical Sciences.

Contributors are required to submit two copies of the manuscripts to the Chief Editor. Manuscripts should be in English and typed in double space. The text of the paper should be preceded by a brief abstract.

References should be listed at the end of the paper in alphabetical order and each reference is to be arranged in the following form:

1. Erdelyi, A., Tables of Integral Transforms, Vol. II, McGraw Hill, (1954)

2 Stokes, G.G., On the effect of the internal friction on the motion of pendulums, Trans. Camb. Phil. Soc., 9 (1851), 8-106.

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Prime Geometric Subsfield

Hom Nath Bhattarai

Introduction

Pasch geometry is an axiomatic system for the structure of double cosets and orbit spaces. This system is developed in [3]. In this short paper we discuss some properties of Pasch geometric sfields, namely, its prime geometric subsfield. We first state some basic definitions and preliminary results.

1. Preliminary

1.1 Definition: By a Pasch geometry is meant a triple $(A, \triangle_A, e)$ where A is a set, $\triangle = \triangle_A \subseteq A \times A \times A$, and $e \in A$ satisfying the following axioms:

I. $\forall$ $a \in A$, there exists a unique $b \in A$ with $(a,b,e) \in \triangle$. Denote b by $a^{\#}$.

II. $e^{\#} = e$ and $(a^{\#})^{\#} = a$ $\forall$ $a \in A$.

III. $(a,b,c) \in \triangle \implies (b,c,a) \in \triangle$.

IV. $(a_1, a_2, a_3), (a_1, a_4, a_5) \in \triangle \implies \exists\, a_6 \in A$ such that $(a_6, a_4^{\#}, a_2), (a_6, a_5, a_3^{\#}) \in \triangle$.

The followings are immediate consequences of I-IV:

V. (i) $(a,b,c) \in \triangle \implies (c^{\#}, b^{\#}, a^{\#}) \in \triangle$.

(ii) $a,b \in A \implies \exists\, c \in A$ with $(a,b,c) \in \triangle$.

In this paper, Geometry will mean Pasch geometry.

1.2 Definition: Let A be a geometry. Then A is called abelian if and only if $(a_1, a_2, a_3) \in \triangle \implies (a_2, a_1, a_3) \in \triangle$; sharp if and only if $(a,b,c), (a,b,d) \in \triangle \implies c = d$ $\forall$ $a,b \in A$; equivalently $(a,a^{\#},b) \in \triangle \implies b = e$ $\forall$ $a \in A$; projective if and only if $a^{\#} = a$ $\forall$ $a \in A$ and $(a,a,b) \in \triangle \implies b = a$ or $b = e$.

1.3 Subgeometry: Let A be a geometry, $S \subseteq A$. Then S is called a subgeometry of A if and only if $e \in S$ and $(s_1, s_2, x) \in \triangle$, $s_1, s_2 \in S$, $x \in A \implies x \in S$. If S is a subgeometry of A, $\triangle_S = \triangle_A \cap (S \times S \times S)$, then $(S, \triangle_S, e)$ is a geometry.

1.4 <u>Factor Geometry</u>: Let S be a subgeometry of a geometry A. For $a, b \in A$, let $a \sim b$ if and only if $\exists\, s_1, s_2 \in S$, $x \in A$ with (a, s_1, x^*), $(x, b^*, s_2) \in \triangle$. Then it is easily seen that $\sim$ is an equivalence relation. For $a \in A$, let $[a]_S = [a] = \{ b \in A / b \sim a \}$. Let $A//_S = \{ [a] / a \in A \}$. Then $[e] = S$. Let $\triangle_{A//_S} = \{ ([a_1], [a_2], [a_3]) : \exists\, x_i \in [a_i]$, $i = 1,2,3$ with $(x_1, x_2, x_3) \in \triangle_A \}$. Then $(A//_S, S, \triangle_{A//_S})$ is a geometry called the factor geometry of A by S.

1.5 <u>Examples</u>: (1) Let G be a group. Let $(a,b,c) \in \triangle_G$ if and only if $abc = 1_G$. Then $(G, 1_G, \triangle_G)$ is a sharp geometry which is abelian if and only if G is abelian group. Suppose A is a sharp geometry. For $a,b \in A$, define $a.b = c^*$ where c is the unique element such that $(a,b,c) \in \triangle$. Then $(A,.)$ is a group.

(2) Let G be a group, H be a subgroup of G. Then the elements in the factor geometry $G//_H$ are the double cosets $\{ HgH : g \in G \}$. $G//_H$ is sharp if and only if H is normal in G.

1.6 <u>Geometric Ring and Sfield:</u> Suppose $(A, o, \triangle_A)$ is a geometry and A is also a monoid with 1_A such that $a.o = o.a = o \ \forall a \in A$. Then A is called a geometric ring if $(a_1, a_2, a_3) \in \triangle$, $a \in A \Longrightarrow (aa_1, aa_2, aa_3) \in \triangle$, $(a_1 a, a_2 a, a_3 a) \in \triangle$. The geometric ring A is called geometric sfield (or skew field) if $A^* = A \setminus \{o\}$ is a group and a geometric field if A^* is an abelian group. A geometric ring A whose geometry is sharp is naturally a ring.

2. <u>The Prime Geometric Subsfield</u>

Let A be a geometric Sfield. Then it can be easily seen that the intersection of any family of geometric subsfields is also a geometric Sfield. The intersection of all the geometric subsfield is also a geometric sfield, which we call the prime geometric subsfield.

Let A be a geometric sfield and R its prime geometric subsfield. Let $\mathcal{Z}(A) = \{ a / a \in A,\ ax = xa \ \forall x \in A \}$. Thus $\mathcal{Z}(A)$ is the centre of the group A^*. The purpose of this paper is to prove that if $R \neq A$, then $R \subseteq \mathcal{Z}(A)$ i.e. R is a geometric field contained in the centre of A.

In [1], we have the following theorem:

2.1 <u>Theorem:</u> (Brauer-Cartan-Hua) Let A be a geometric Sfield and B be a geometric subsfield of A. Then $B \neq A$, $B^* \trianglelefteq A^* \Longrightarrow B \subseteq \mathcal{Z}(A)$.

From the above theorem, it is clear that all we need prove is that if R is prime geometric subsfield, $R \neq A$, then $R^* \trianglelefteq A^*$. We begin with a series of results.

Let A be a geometry and $X \subseteq A$. An intersection of all subgeometries of A containing X is also a subgeometry of A, which we denote by $\langle X \rangle$

2.2 Proposition: Let A be an abelian geometry and let $x \in A$. Let $(x)_o = \{e\}$, $(x)_1 = \{y: (y, x^{\epsilon_1}, x^{\epsilon_2}) \in \Delta,\ x^{\epsilon_i} = x \text{ or } x^{\#},\ i = 1,2\}$ and by induction, let $(x)_n = \{z: (z, y_1, y_2) \in \Delta,\ y_1, y_2 \in (x)_{n-1}\}$. Then, $\langle x \rangle = \bigcup_{n=o}^{\infty} (x)_n$.

Proof: Since $\langle x \rangle$ is a subgeometry, $e \in \langle x \rangle$ so $(x)_o \subseteq \langle x \rangle$. Also, $x, x^{\#} \in \langle x \rangle$ hence $(x)_1 \subseteq \langle x \rangle$. Now, by induction $(x)_n \subseteq \langle x \rangle$ for all n. Thus $\bigcup_{n=o}^{\infty} (x)_n \subseteq \langle x \rangle$. Therefore, it suffices to show that $\bigcup_{n=o}^{\infty} (x)_n$ is a subgeometry. Note that $(x)_i \subseteq (x)_j$ for $i \leq j$. Let $(x_1, x_2, z) \in \Delta$, $x_1, x_2 \in \bigcup_{n=o}^{\infty} (x)_n$. Then, $x_1, x_2 \in (x)_m$ for some m. So $(z, x_1, x_2) \in \Delta \Longrightarrow z \in (x)_{m+1} \subseteq \bigcup_{n=o}^{\infty} (x)_n$. Thus $\bigcup_{n=o}^{\infty} (x)_n$ is a subgeometry.

2.3 Proposition: Let A be an abelian geometry. Let $X \subseteq A$. Let $\mathcal{S} = \{\langle S \rangle : S \subseteq X, S \text{ is finite}\}$. Then $\langle x \rangle = \cup \mathcal{S}$

Proof: It suffices to show that $\cup \mathcal{S}$ is a subgeometry. Let $(z, x_1, x_2) \in \Delta$, $x_1, x_2 \in \cup \mathcal{S}$. Then $x_1 \in \langle S \rangle$, $x_2 \in \langle T \rangle$ where $S \subseteq X$, $T \subseteq X$, S, T are finite, then, $z \in \langle S \cup T \rangle$. But $S \cup T$ is finite, so $\langle S \cup T \rangle \in \mathcal{S}$. Thus $z \in \cup \mathcal{S}$. Hence $\cup \mathcal{S}$ is a subgeometry.

2.4. Lemma: Let A be abelian geometry. Then:

(i) $(x_1, s_1, t_1), (x_2, s_2, t_2), (x_1, x_2, x_3) \in \Delta \Longrightarrow \exists\, s_3, t_3 \in A$ such that $(s_1, s_2, s_3), (t_1, t_2, t_3), (s_3, t_3, x_3) \in \Delta$.

(ii) If S_1, S_2 are subgeometries, then $\langle S_1 \cup S_2 \rangle = \{a : a \in A,\ (a, s_1, s_2) \in \Delta \text{ for some } s_1 \in S_1, s_2 \in S_2\}$.

Part (i) follows easily and (ii) can be proved easily by using (i).

Let A be a geometric Sfield, $X \subseteq A$. Let $\overline{X}$ denote the subgroup of A^* generated by X^* together with 0. Let $[x]$ denote the smallest geometric Sfield generated by X. The proof of the following lemma is similar to 2.2.

2.5 Lemma: Let $X \subseteq A$, $S_o = \langle X \rangle$, $S_i = \langle \overline{S}_{i-1} \rangle$ defined inductively for $i = o, 1, 2, \ldots\ldots$ Then, $[X] = \bigcup_{i=o}^{\infty} S_i$.

2.6 Lemma: Let $S \subseteq A$, where A is a geometric Sfield. Then $aSa^{-1} \subseteq S$ $\forall\, a \in A^* \Longrightarrow [S]^* \trianglelefteq A^*$.

Proof: Let $a \in A^*$. We first show that if $X \subseteq A$, with $aXa^{-1} \subseteq X$, then $a \langle X \rangle a^{-1} \subseteq \langle X \rangle$. By (2.3), $\langle X \rangle = \cup \{\langle T \rangle : T \subseteq X, T \text{ is finite}\}$. So we may assume X finite. Induct on $|X|$, the cardinality of X. If

$X = \{x\}$, then by (2.2), $\langle x \rangle = \bigcup_{n=0}^{\infty} (x)_n$. Let $y \in \langle x \rangle$. Then $y \in (x)_i$. If $i=1$, then $(y, x^{\epsilon_1}, x^{\epsilon_2}) \in \triangle$. So $(aya^{-1}, (axa^{-1})^{\epsilon_1}, (axa^{-1})^{\epsilon_2}) \in \triangle$. So $aya^{-1} \in \langle x \rangle$. If $i \neq 1$, $(y, z_1, z_2) \in \triangle$, $z_1, z_2 \in (x)_{i-1}$. So $(aya^{-1}, az_1a^{-1}, az_2a^{-1}) \in \triangle$. By induction, $aya^{-1} \in a(x)_i a^{-1} \subseteq \langle x \rangle$. Now, if $X = \{x_1, x_2, \ldots, x_n\}$, then $\langle X \rangle = \langle \langle x_1, \ldots x_{n-1} \rangle \cup \langle x_n \rangle \rangle$. So by (2.4 ii), $(y, z_1, z_2) \in \triangle$, $z_1 \in \langle x_1, \ldots, x_{n-1} \rangle$, $z_2 \in \langle x_n \rangle$. So $(aya^{-1}, az_1a^{-1}, az_2a^{-1}) \in \triangle$. By induction $az_1a^{-1}, az_2a^{-1} \in \langle x \rangle$. Thus, $a \langle X \rangle a^{-1} \subseteq \langle X \rangle$. Now, $[S] = \bigcup_{i=0}^{\infty} S_i$. $S_o = \langle S \rangle$, so $aS_oa^{-1} \subseteq [S]$. Suppose $aS_ia^{-1} \subseteq [S]$. Then clearly $a\overline{S}_ia^{-1} \subseteq [S]$, so from above $aS_{i+1}a^{-1} = a \langle \overline{S}_i \rangle a^{-1} \subseteq S$. Hence $[S]^* \trianglelefteq A^*$.

2.7 <u>Theorem:</u> Let A be a geometric Sfield and R be the prime geometric subsfield. If $R \neq A$, then $R \subseteq Z(A)$.

<u>Proof:</u> We note that $R = [1]$, the geometric subsfield generated by 1. Since $a^{-1}.1.a = 1 \in \{1\}$ by lemma 2.5, we have $R^* \trianglelefteq A^*$. So by 2.1, $R \subseteq Z(A)$.

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Mathematics Instruction Committee
Kirtipur Multiple Campus
Tribhuvan University
NEPAL.

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Connections in the Tangent Bundle T (M^n)

R.S. Mishra and J.P. Srivastava

1. Abstract

In the present paper, we have determined Intermediate lift ∇^I on $T(M^n)$ of a torsion free linear connection ∇ on M^n and some properties with ∇^I.

2. Introduction

The indices a, b, c, i, j, ... have range in $(1,\ldots,n)$ while indices A, B, C, λ, μ, ν ... have range in $(1,\ldots n,n+1,\ldots 2n)$ we put $\bar{i} = i+n$. Summation over repeated indices is always implied. For the notations and definitions we usually follow [3].

Let M^n be a C - manifold of dimensions n. $T(M^n)$ is its tangent bundle of dimensions 2n. Let V be the field of n planes tangent to the fibres of $T(M^n)$. This is an integrable vertical distribution. A torsion - free linear connection on M^n determines uniquely in $T(M^n)$ an n-dimensional horizontal distribution H complementary to V. The pair (H,V) defines an almost product structure in $T(M^n)$ and turns it into an almost product manifold.

The horitzontal distribution H is spanned by n-independent vector fields [2,3].

$$D_j = \frac{\partial}{\partial x^i} - \Gamma_j^h \frac{\partial}{\partial y^h} \tag{1.1}$$

where.

$$\Gamma_j^h = \Gamma_{ji}^h y^i$$

The vertical distribution V is spanned by the n-independent vector fields.

$$D_{\bar{j}} = \frac{\partial}{\partial y^j} \tag{1.2}$$

Here $(D_\lambda) = \{D_j, D_{\bar{j}}\}$ constitute a frame on $T(M^n)$ which is called adapted frame on $T(M^n)$. Components of tensors and geometrical objects with respect to the frame are called frame components.

Note: The number in brackets refer to References at the end of the paper.

The coframe dual to the adapted frame are

$$\{\theta^j, \theta^{\bar{j}}\}$$

where

(1.3) $\quad \theta^j = dx^j$

and

(1.4) $\quad \theta^{\bar{j}} = dy^j + \Gamma_i^j \, dx^i$

The non-holomomic object $\Omega^{\nu}_{\lambda\mu}$ of the adapted frame are defined by

(1.5) $\quad [D_\lambda, D_\mu] = \Omega^{\nu}_{\lambda\mu} D_\nu ,$

and non-vanishing components of which are [3].

$$(1.6) \quad \begin{aligned} \Omega_{ji}^{\bar{h}} &= - \Omega_{ij}^{\bar{h}} = - R_{jik}^{\ \ h} y^k , \\ \Omega_{j\bar{i}}^{\bar{h}} &= - \Omega_{\bar{i}j}^{\bar{h}} = \Gamma_{ji}^{h} ; \end{aligned}$$

where $R_{jik}^{\ \ h}$ being components of the curvature tensor R of a torsion-free linear connection ∇ on M^n.

Let $\tilde{\nabla}$ be an arbitrary linear connection on $T(M^n)$ whose components are $\tilde{\Gamma}^A_{CB}$. The frame components of $\tilde{\nabla}$ are defined by

(1.7) $\quad \tilde{\Gamma}^{\nu}_{\lambda\mu} = (D_\lambda (L^A_\mu) + \tilde{\Gamma}^A_{CB} L^C_\lambda L^B_\mu) L^{\nu}_A ,$

where

$$L^A_\mu = \begin{bmatrix} \delta^h_i & 0 \\ -\Gamma^h_i & \delta^h_i \end{bmatrix} \quad \text{and} \quad L^{\nu}_A = \begin{bmatrix} \delta^h_i & 0 \\ \Gamma^h_i & \delta^h_i \end{bmatrix}$$

The frame components of the curvature tensor $\tilde{R}$ of $\tilde{\nabla}$ are given by

(1.8) $\quad \tilde{R}^{\nu}_{\omega\lambda\mu} = D_\omega \tilde{\Gamma}^{\nu}_{\lambda\mu} - D_\lambda \tilde{\Gamma}^{\nu}_{\omega\mu} + \tilde{\Gamma}^{\nu}_{\omega\sigma} \tilde{\Gamma}^{\sigma}_{\lambda\mu} - \tilde{\Gamma}^{\nu}_{\lambda\sigma} \tilde{\Gamma}^{\sigma}_{\omega\mu} - \Omega^{\sigma}_{\omega\lambda} \tilde{\Gamma}^{\nu}_{\sigma\mu}$

The conditions of parallelism for h and V with respect to a linear connection $\tilde{\nabla}$ on $T(M^n)$ are [1,2].

$$
(1.9)\qquad
\begin{aligned}
&H \text{ is path parallel iff } \tilde{\Gamma}_{ji}^{\bar{h}} + \tilde{\Gamma}_{ij}^{\bar{h}} = 0,\\
&H \text{ is parallel along } V \text{ iff } \tilde{\Gamma}_{ji}^{\bar{h}} = 0,\\
&H \text{ is parallel iff } \tilde{\Gamma}_{\lambda i}^{h} = 0,\\
&V \text{ is path parallel iff } \tilde{\Gamma}_{\bar{j}\bar{i}}^{h} + \tilde{\Gamma}_{\bar{i}\bar{j}}^{h} = 0,\\
&V \text{ is parallel along } H \text{ iff } \tilde{\Gamma}_{j\bar{i}}^{h} = 0,\\
&V \text{ is parallel iff } \tilde{\Gamma}_{\lambda\bar{i}}^{h} = 0.
\end{aligned}
$$

here all parallelisms are with respect to $\tilde{\nabla}$ and $\tilde{\Gamma}_{\lambda\mu}^{\nu}$ are the fame components of $\tilde{\nabla}$.

Now, we will consider A-tensor and B-tensor, tensor of type (1.2) associated with the linear connection $\tilde{\nabla}$ on $T(M^n)$. The frame components of which are

$$
(1.10)\qquad
\begin{aligned}
&A_{ji}^{h} = 0,\ A_{ji}^{\bar{h}} = \tilde{\Gamma}_{ji}^{\bar{h}},\ A_{j\bar{i}}^{h} = \tilde{\Gamma}_{j\bar{i}}^{h},\ A_{j\bar{i}}^{\bar{h}} = 0\\
&A_{\bar{j}i}^{h} = 0,\ A_{\bar{j}i}^{\bar{h}} = \tilde{\Gamma}_{\bar{j}i}^{\bar{h}},\ A_{\bar{j}\bar{i}}^{h} = \tilde{\Gamma}_{\bar{j}\bar{i}}^{h},\ A_{\bar{j}\bar{i}}^{\bar{h}} = 0,\\
&B_{ji}^{h} = 0,\ B_{ji}^{\bar{h}} = -\tfrac{1}{2}\tilde{\Gamma}_{ji}^{\bar{h}},\ B_{j\bar{i}}^{h} = -\tilde{\Gamma}_{j\bar{i}}^{h},\\
&B_{j\bar{i}}^{\bar{h}} = -\tilde{\Gamma}_{\bar{i}j}^{\bar{h}},\ B_{\bar{j}i}^{h} = -\tilde{\Gamma}_{i\bar{j}}^{h},\ B_{\bar{j}i}^{\bar{h}} = -\tilde{\Gamma}_{\bar{j}i}^{\bar{h}}\\
&B_{\bar{j}\bar{i}}^{h} = -\tfrac{1}{2}(\tilde{\Gamma}_{\bar{j}\bar{i}}^{h} + \tilde{\Gamma}_{\bar{i}\bar{j}}^{h}),\ B_{\bar{j}\bar{i}}^{\bar{h}} = 0.
\end{aligned}
$$

Thus we have

Lemma (1.1). Let $\tilde{\nabla}$ be a torsion free linear connection in $T(M^n)$ and B its associated B-tensor. With respect to the linear connection $\tilde{\nabla}$ + B, H is parallel along V, V is parallel along H and both H and V are path parallel.

Lemma (1.2). Let $\tilde{\nabla}$ be a torsion free linear connection in $T(M^n)$ and A its associated A tensor. With respect to the linear connection $\tilde{\nabla}$ - A, both H and V are parallel.

2. Complete lift of a linear connection

Let ∇ be a linear connection in M^n. In [3] we find g^c on $T(M^n)$, namely the complete lift of g. The components of g^c are

$$(2.1) \qquad g^c = \begin{bmatrix} y^k \partial_k g_{ij} & g_{ij} \\ g_{ij} & 0 \end{bmatrix}$$

where $g = (g_{ij})$ is the Riemannian metric on M^n. The frame components of g^c are

$$(2.2) \qquad \begin{bmatrix} 0 & g_{ij} \\ g_{ij} & 0 \end{bmatrix}$$

Let ∇^c be the complete lift of ∇ and $\tilde{\Gamma}^{\nu}_{\lambda\mu}$ be the frame components of ∇^c. Then we have

$$(2.3) \qquad \tilde{\Gamma}^h_{ji} = \Gamma^h_{ji}, \quad \tilde{\Gamma}^{\bar{h}}_{ji} = -R^h_{jik}\, y^k, \quad \tilde{\Gamma}^{\bar{h}}_{j\bar{i}} = \Gamma^h_{ji}.$$

With help of (2.3) and (1.8), we obtain the possibly nonzero frame components of the curvature $\tilde{R}$ of ∇^c are

$$(2.4) \qquad \begin{aligned} &\tilde{R}^h_{kji} = R^h_{kji}, \quad \tilde{R}^{\bar{h}}_{kji} = y^a \nabla_a R^h_{kji}, \quad \tilde{R}^{\bar{h}}_{kj\bar{i}} = R^h_{kji}, \\ &\tilde{R}^{\bar{h}}_{k\bar{j}i} = R^h_{kji}, \quad \tilde{R}^{\bar{h}}_{\bar{k}ji} = R^h_{kji}. \end{aligned}$$

we have

Proposition (2.1). Let ∇^c be the complete lift on $T(M^n)$ of a torsion free linear connection ∇ in M^n. Then $(T(M^n), \nabla^c)$ is locally flat iff (M^n, ∇) is locally flat.

Further, we compute the frame components of $\nabla^c \tilde{R}$. Its possibly nonzero frame components are

$$(2.5) \qquad \begin{aligned} &\nabla^c_\lambda \tilde{R}^h_{kji} = \nabla_\lambda R^h_{kji}, \quad \nabla^c_\lambda \tilde{R}^{\bar{h}}_{kji} = y^a \nabla_\lambda \nabla_a R^h_{kji}, \\ &\nabla^c_\lambda \tilde{R}^{\bar{h}}_{kj\bar{i}} = \nabla^c_\lambda \tilde{R}^{\bar{h}}_{k\bar{j}i} = \nabla^c_{\bar{\lambda}} \tilde{R}^{\bar{h}}_{kji} = \nabla_\lambda R^h_{kji} \end{aligned}$$

we have

Proposition (2.2). $(T(M^n), \nabla^C)$ is locally symmetric iff (M^n, ∇) is locally symmetric.

3. horizontal lift of a linear connection

In [3] the components of ∇^H have been found and they are

$$\tilde{\Gamma}_{ji}^{h} = 0,\ \tilde{\Gamma}_{j\bar{i}}^{h} = 0,\ \tilde{\Gamma}_{\bar{j}i}^{h} = 0,\ \tilde{\Gamma}_{\bar{j}\bar{i}}^{h} = 0\quad \tilde{\Gamma}_{ji}^{\bar{h}} = \partial\Gamma_{ji}^{h} - R_{kji}^{h} y^{k}$$

$$(3.1)\qquad \tilde{\Gamma}_{j\bar{i}}^{\bar{h}} = \Gamma_{ji}^{h},\ \tilde{\Gamma}_{\bar{j}i}^{\bar{h}} = \Gamma_{ji}^{h},\ \Gamma_{\bar{j}\bar{i}}^{\bar{h}} = 0.$$

Also, computing its frame components, we find its possibly nonzero frame components as

$$(3.2)\qquad \tilde{\Gamma}_{ji}^{h} = \Gamma_{ji}^{h},\ \tilde{\Gamma}_{ji}^{\bar{h}} = \Gamma_{ji}^{h}$$

With help of (3.2) and (1.8) we obtain the possibly nonzero frame components of the curvature tensor $\tilde{R}$ of ∇^H and $\nabla^H\tilde{R}$ as

$$(3.3)\qquad \tilde{R}_{kji}^{h} = R_{kji}^{h},\ \tilde{R}_{kj\bar{i}}^{\bar{h}} = R_{kji}^{h}$$

$$(3.4)\qquad \nabla^{H}_{\ell}\tilde{R}_{kji}^{h} = \nabla_{\ell}R_{kji}^{h},\ \nabla^{H}_{\ell}R_{kj\bar{i}}^{\bar{h}} = \nabla_{\ell}R_{kji}^{h}$$

Then, we have

Proposition (3.1). Let ∇^H be the horizontal lift on $T(M^n)$, of a torsion free linear connection ∇ in M^n. Then $(T(M^n), \nabla^H)$ is locally flat (resp, locally symmetric) iff (M^n, ∇) is locally flat (resp, locally symmetric).

4. Intermediate lift ∇^I of ∇

Now, we construct from ∇^C a linear connection ∇^I on $T(M^n)$. For this purpose we obtain the frame-components of the B-tensor B associated with ∇^C we find that only possibly nonzero frame-components are

$$(4.1)\qquad B_{ji}^{\bar{h}} = -\tfrac{1}{2}\, R_{jik}^{h}\, y^{k}$$

Again, we get the possibly nonzero frame components of the linear connection $\nabla^C + B$ as

$$(4.2)\quad \tilde\Gamma_{ji}^{h} = \Gamma_{ji}^{h}, \quad \tilde\Gamma_{ji}^{\bar h} = -\tfrac{1}{2} R_{jik}^{\;\;h} y^k, \quad \tilde\Gamma_{j\bar i}^{\bar h} = \Gamma_{ji}^{h},$$

Then we have

Proposition (4.1). Let $\nabla^I = \nabla^C + B$ be the linear connection on $T(M^n)$. Then with respect to ∇^I, H is parallel along V, V is parallel along H and H and V are path parallel.

By comparing the equations (4.2) and (1.9), we notice that V is parallel in $(T(M^n), \nabla^I)$, but H is not parallel. H will be parallel if ∇ is locally flat.

Again the possibly nonzero frame components of the A-tensor A associated with ∇^I are

$$(4.3)\qquad A_{ji}^{\bar h} = -R_{jik}^{\;\;h} y^k$$

The linear connection $\nabla^I - A$ has possibly nonzero frame components as

$$(4.4)\qquad \tilde\Gamma_{ji}^{h} = \Gamma_{ji}^{h}, \quad \tilde\Gamma_{j\bar i}^{h} = \Gamma_{ji}^{h}$$

But this is exactly the same as in (3.2).

Proposition (4.2). We have $\nabla^I = \nabla^H + A$.

Now, we shall calculate the curvature tensor $\tilde R$ of ∇^I. Its possibly nonzero frame components are

$$(4.5)\quad \begin{aligned} &\tilde R_{kji}^{\;\;\;h} = R_{kji}^{\;\;\;h}, \quad \tilde R_{kji}^{\;\;\;\bar h} = \tfrac{1}{2} y^a \nabla_a R_{kji}^{\;\;\;h}, \\ &\tilde R_{kj\bar i}^{\;\;\;\bar h} = R_{kji}^{\;\;\;h}, \quad \tilde R_{k\bar j i}^{\;\;\;\bar h} = -\tilde R_{\bar j k i}^{\;\;\;\bar h} = \tfrac{1}{2} R_{ikj}^{\;\;\;h} \end{aligned}$$

Thus, we have

Proposition (4.3). Let ∇^I be an intermediate lift on $T(M^n)$ of a torsion-free linear connection ∇ on M^n. Then $(T(M^n), \nabla^I)$ is locally flat iff (M^n, ∇) is locally flat.

Furthermore, the possibly nonzero frame components of $\overset{I}{\nabla}\tilde{R}$ are

$$\overset{I}{\nabla}_{\lambda}\tilde{R}_{kji}{}^{h} = \nabla_{\lambda}R_{kji}{}^{h}, \quad \overset{I}{\nabla}_{\lambda}\tilde{R}_{kji}{}^{\bar{h}} = \tfrac{1}{2}\, y^{a}\,\nabla_{\lambda}\nabla_{a}\, R_{kji}{}^{h},$$

$$\overset{I}{\nabla}_{\lambda}\tilde{R}_{kj\bar{i}}{}^{\bar{h}} = \nabla_{\lambda}R_{kji}{}^{h}, \quad \overset{I}{\nabla}_{\lambda}\tilde{R}_{k\bar{j}i}{}^{\bar{h}} = \tfrac{1}{2}\nabla_{\lambda}R_{ikj}{}^{h},$$

$$\overset{I}{\nabla}_{\bar{\lambda}}\tilde{R}_{kji}{}^{h} = \nabla_{\lambda}R_{kji}{}^{h}.$$

Then,

Proposition (4.4). $(T(M^{n}), \overset{I}{\nabla})$ is locally symmetric iff (M^{n}, ∇) is locally symmetric.

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Department of Mathematics
Banaras Hindu University
Varanasi-5
India.

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The Skin Friction on Infinite Cylinders Parallel to Their Lengths in Walters Liquid B'

R.P. Gupta & B.M. Gaur

In the present paper, we propose to study the three dimensional unidirectional motion of elastico-viscous Walters Liquid B' [4] generated by the forced motion of an infinite cylinder parallel to its length. It is supposed that the cylinder and the fluid are at rest initially and the cylinder is given a velocity W, which remains steady thereafter.

With this choice the equation of motion assumes the form

$$(1) \qquad \frac{\partial w}{\partial t} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \frac{k_o^*}{r}\frac{\partial^2}{\partial t\,\partial r}\left(r\frac{\partial w}{\partial r}\right)$$

where ν and K_o^* are viscous and elastic parameters, respectively and w the velocity parallel to the generators of the cylinder. The equation (1) is to be solved under following boundary conditions

(2)
(a) $w = o$, everywhere in the fluid at $t = o$, $r = a$.
(b) $w = W$, a constant on the boundary of the cylinder for all $t > o$.

As k_o^* is a small quantity (cf. Beard & Walters [2]), we solve equation (1) for small values of k_o^* by substituting $w = w_o + k_o^* w_1$ in it and neglecting squares of k_o^*, thus we obtain,

$$(3) \qquad \frac{\partial w_o}{\partial t} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(\frac{r\,\partial w_o}{\partial r}\right),$$

$$(4) \qquad \frac{\partial w_1}{\partial t} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(\frac{r\,\partial w_1}{\partial r}\right) - \frac{1}{r}\frac{\partial^2}{\partial t\,\partial r}\left(r\frac{\partial w_o}{\partial r}\right).$$

The solution of equation (3) has been given by Batchelor [1], utilising the analogy with the corresponding heat conduction problem discussed earlier by Carslaw and Jaeger [3]. Our aim is to investigate the modifications by the elastic parameter.

The solution of equation (3) as given in Carslaw and Jaeger is

$$(5) \quad w_o = W + 2W \int_o^{\infty} e^{-k^2 t} \; J_o(kr)Y_o(ka) - J_o(ka)Y_o(kr) \; dk \,,$$

where r is the radial measure from the axis of cylinder, a is the radius of the cylinder J_o and Y_o are the Bessels's functions of order zero.

The solution of equation (4) under conditions $w_1 = o$ everywhere at $t = o$ and $w_1 = o$ at $r = a$, $t > o$, is

$$(6)\quad w_1 = -2\nu Wt \int_0^\infty k^3 e^{-k^2\nu t} \left\{ \frac{J_o(kr)Y_o(ka) - J_o(ka)Y_o(kr)}{J_o^2(ka) + Y_o^2(ka)} \right\} \cdot dk,$$

Calculation of Skin Friction

It is worth discussion of the elastic effect on the frictional force on the unit area of the cylinder due to the forced motion of the liquid. Contribution of viscous force calculated by Batchelor [1] is

$$C_o|(\nu t/a^2) = \frac{4}{\pi^2} \int_0^\infty \frac{e^{-k^2\nu t}}{J_o^2(ka) + Y_o^2(ka)} \frac{dk}{k},$$

and, the corresponding elastic frictional force on the unit area of the cylinder is calculated here and comes to be

$$(7)\quad C_1(\nu t/a^2) = \frac{a^3}{\nu W} [\nu \frac{\partial w_1}{\partial r} - \frac{\partial^2 w_o}{\partial t \partial r}] = \frac{4\nu t}{\pi^2} \int_0^\infty \frac{k(k^2\nu t + 1)}{J_o^2(ka) + Y_o^2(ka)} dk.$$

If we follow the analysis of Carslaw and Jaeger to calculate the elastic part of the skin friction for small values of time, a term of $O(T)^{-3/2}$ will appear which shows that the elastic forces are the dominant forces, therefore, in this case, that analysis is not applicable for small times, since the perturbed elastic part should not be more significant than the initial viscous part.

Now we will find the skin friction for large values of time. Carrying out the analysis exactly as in [3], we find following expression for the skin friction on the unit area of the cylinder,

$$(8)\qquad C = C_o + K_o^* C_1,$$

where the viscous part C_o as given by Batchelor is

$$C_o = 2\left[\frac{1}{\log 4T - 2\gamma} - \frac{2\gamma}{(\log 4T - 2\gamma)^2} + \ldots\right]$$

and the elastic part obtained here is

$$C_1 = \frac{8}{T[\log 4T - 2\gamma]}\left[\frac{1}{\log 4T - 2\gamma} + \frac{2\gamma}{(\log 4t - 2\gamma)^2} + \ldots\right]$$

The following table for C/C_o shows how the skin friction increases on account of elasticity but the effect decreases with increasing time.

LOG T	C/C_o		
	$K_o^* = 0.2$	$K_o^* = 0.4$	$K_o^* = 0.6$
1	1.23160	1.39710	1.53160
2	1.13970	1.27940	1.21922
3	1.04820	1.04964	1.07446
4	1.00580	1.01168	1.01750
5	1.00160	1.00320	1.00480
6	1.00045	1.00090	1.00135
7	1.00013	1.00024	1.00041
8	1.00004	1.00007	1.00011
9	1.00002	1.00002	1.00004
10	1.00000	1.00001	1.00001

Sincere thanks are due to Dr. S. Datta for his kind guidance during preparation of this paper

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Department of Mathematics and Astronomy
Lucknow University
Lucknow (India)

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On the Minimum of the (C,) Graph of the Dispersion Curve

$$c^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}\right) \text{th} \frac{2\pi h}{\lambda}.$$

M.P. Sinha and S.R.P. Sinha

Abstract

An analysis of the (c,λ)- equation $c^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}\right) \text{th} \frac{2\pi h}{\lambda}$ has been presented here. It has been found that its graph has a single minimum if and only if $h > \sqrt{3T/\rho g}$. The (c,) curve has been traced and its position relative to that for the liquid of infinite depth has been indicated.

Introduction

The (c,λ)- relation in the propagation of two-dimensional progressive periodic waves on the free surface of a heavy homogeneous liquid of constant depth h is given by

$$(1) \qquad c^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}\right) \text{th} \frac{2\pi h}{\lambda}, \qquad \ldots \ldots \ldots \quad (C)$$

where T denotes the surface tension at the free surface and ρ the density of the liquid. For a liquid of inifinite depth (i.e. $h/\lambda \to \infty$) the above relation reduces to the simple form,

$$(2) \qquad c^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}\right). \qquad \ldots \ldots \ldots \quad (G_1)$$

This simple case has been discussed in detail in the Lamb's Hydrodynamics [2, § 267]. It has been found that the curve (G_1) has a single extremum, which is a minimum, at $\lambda = 2\pi\sqrt{T/\rho g}$ and the minimum value of c is $(4gT/\rho)^{\frac{1}{4}}$. Because of the term $\text{th}\frac{2\pi h}{\lambda}$ in the right hand side the dispersion curve (C) may not have a minimum for every value of h. However, in view of the fact that it has minimum for $h \to \infty$, one may hope that there is a critical value of h, say h_c, such that (C) has a minimum whenever h h_c. We have found in this article that $h_c = \sqrt{(3T/\rho g)}$ as against $\sqrt{(3T/2\rho g)}$ quoted in the Encyclopaedia of physics [1, pp. 633 and 515] in several connections without giving any details of calculation.

1. Asymptotes of the (c,λ) - graph of (C):

The (c,λ)- graph of (C) is symmetric about the origin. But as we are interested in the positive value of c and λ we shall trace it in the positive-quadrant of the (c,λ) - plane.

From (1) we find that the curve (C) has asymptotes $\lambda = 0$ and $c = \sqrt{gh}$. Accordingly, the curve (C) will have the following two possible graphs;

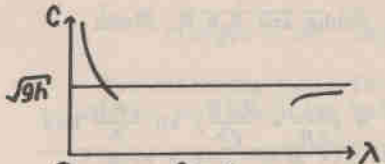

fig. 1

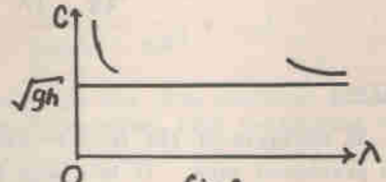

fig. 2

In the first case the curve is asymptotic to $c = \sqrt{gh}$ from below, whereas in the second from above. In the first possibility the curve will attain an odd number of extremum values, whereas in the second it will have either no extremum or has an even number of them before being tangent to $c = \sqrt{gh}$ at infinity. We, therefore, infer that if (C) has a single extremum it must be minimum, and also in this case, it will cross the line $c = \sqrt{gh}$ to attain the minimum before proceeding to meet it again asymptotically. If the curve (C) has either no extremum or an even number of them, it does not cross $c = \sqrt{gh}$.

We shall show in the following that (C) has a single extremum and it crosses the line $c = gh$ if and only if $h > \sqrt{(\frac{3T}{g})}$.

2. Condition for a single extremum of (C):

The curve can be written as

$$(1)' \qquad c^2 = g\left(x + \frac{a^2}{x}\right) \operatorname{th} \frac{h}{x}\,.$$

$$\text{where} \quad x = \frac{\lambda}{2\pi} \quad \text{and} \quad a^2 = \frac{T}{\rho g}\,.$$

Since the physical quantities involved must be positive, we may discuss the extremum value of c^2 instead of c. For an extremum value c^2 we must have

$$\frac{dc^2}{dx} = \frac{gh}{x^3}\left[(x^2 - a^2)\,\frac{\operatorname{sh}(\frac{2h}{x})}{\frac{2h}{x}} - (x^2 + a^2)\right] \operatorname{sech}^2\left(\frac{h}{x}\right) = 0.$$

It is obvious that an extremum point should satisfy the inequality $x^2 > a^2$ and is given by the transcedental equation

$$(3) \qquad (x^2 + a^2)/(x^2 - a^2) = \operatorname{sh}(2h/x)/(2h/x)$$

The above equation seems to be intractable for the quantitative determination of the roots. However, we may look for their existence qualitatively by graphic methods.

The extremum points will be points of intersection of the two curves;

(4) $$y = \frac{x^2 + a^2}{x^2 - a^2} \quad \dots \dots \dots \quad (C_1)$$

(5) $$y = sh(2h/x) \; / \; (2h/x) \quad \dots \dots \dots \quad (C_2)$$

We note that the curve (C_1) is symmetrical about the y-axis and has asymptote $y = 1$, $x = \pm a$. Also, since

$$\frac{dy}{dx} = -\frac{4xa^2}{(x^2-a^2)^2} \quad \text{and} \quad \frac{d^2y}{dx^2} = 4a^2 \; (a^2+3x^2)/(x^2-a^2)^3,$$

y and $\frac{dy}{dx}$ decrease monotonically in the interval $a < x < +\infty$.

The curve (C_2) is symmetrical about the y-axis and wholly lies in the upper half, $y > 0$, of the (x,y)-plane. Also, since

$$\frac{dy}{dx} = \frac{1}{2h} \cdot \; ch \frac{2h}{x} \; (th \frac{2h}{x} - \frac{2h}{x}) \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{4h^2}{x^3} \; sh \frac{2h}{x},$$

y decreases and $\frac{dy}{dx}$ increases in the interval $0 < x < +\infty$.

Moreover (C_2) is asymptotic to $y = 1$ and $x = 0$. From these, it will not be difficult to infer that (C_1) and (C_2) can have one of the following two modes of intersection:

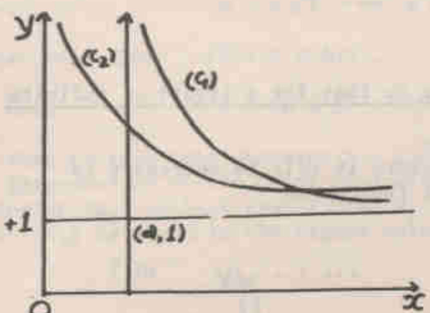

fig.3

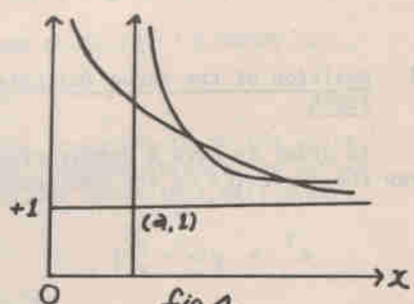

fig.4

The situation given in the Fig. 4 is ruled out if and only if the curve (C_1) falls below the curve (C_2) for a big value of x, in which case there will be a single intersection point. For this we write the asymptotic expansions of (C_1) and (C_2), for a great x, as follows:

(C) $$y = 1 + \frac{2a^2}{x^2} + \frac{2a^4}{x^4} + \quad \dots \quad \dots$$

$$\text{(C)} \qquad y = 1 + \frac{2h^2}{3x^2} + \frac{2h^4}{15x^2} + \quad \dots \quad \dots$$

From here we note that, for a great x, $y_{(c_2)} > y_{(c_1)}$ whenever $h^2 = 3a^2$. Also for $h^2 = 3a^2$, we easily find that $y_{(c_2)} < y_{(c_1)}$ for a big value of x.

Thus the curve (C) has a single extremum if and only if $h > a\sqrt{3}$ i.e. $h > \sqrt{\frac{3T}{\rho q}}$. On the basis of the analysis put forward in § 1, this extremum will be the minimum of (C). One may verify once again by graphic method that the curve (C) crosses its asymptote $c = \sqrt{gh}$ whenever $h > \sqrt{(3T/\rho g)}$. In this case the curve (C) will have the following rough sketch,

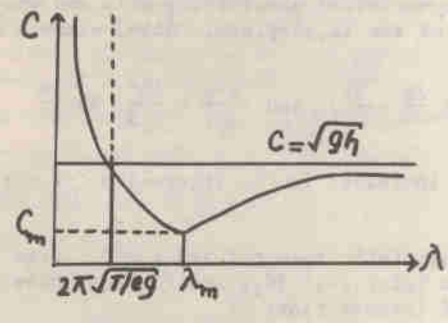

fig. 5

3. <u>Position of the curve (C) relative to that for a liquid of infinite depth</u>

In order to have a comparative picture it will be necessary to draw the curve (C) on the background of the curves

$$c^2 = g(x + \frac{a^2}{x}) \qquad \dots \dots \dots \qquad (G_1)$$

$$c^2 = gx \qquad \dots \dots \dots \qquad (G_2)$$

$$c^2 = g\,\frac{a^2}{x} \qquad \dots \dots \dots \qquad (G_3)$$

where (G_1) is the dispersion-relation for periodic progressive waves on a liquid of infinite depth under the combined effect of gravity and surface tension; (G_2) and (G_3) are the relation for purely gravity and purely capillsry waves respectively.

The curve (G_1) has been drawn and discussed in detail in the Lamb's Hydrodynamics [2, § 267]. It is asymptotic to the c axis and has the minimum at $x = a$, and $c_{min} = \sqrt{2ag}$. Let us denote this point by M. We also note that the curve (G_1) meets the curve (C_1) only at $(x = 0, c = \infty)$, where both the curves are asymptotic to the c axis. It is, therefore, evident that the curve (C) will always be below (G_1) lest it will again intersect with (G_1) before being asymptotic to the line $c = \sqrt{gh}$. The curves (G_2) and (G_3) intersect at $(x = a, c = \sqrt{ga})$. We denote this point by A. We find that the line AM is parallel to the c-axis. Let us denote the points of intersection of the line AM with the curve (C) and the line $c = \sqrt{gh}$ by L and B respectively. We shall show that the curve (C) is always above the line $c = \sqrt{ga}$, whenver, it has the minimum. In other words, the minimum value of c is greater than $\sqrt{ga}$. We have already noted in § 2 that the minimum point of the curve (C) lies on the right hand side of the line AM. In order to show that the curve (C) always lies above the line $c = \sqrt{ga}$, it will be sufficient to prove that the line has no real intersection with the curve (C). For this, we show that the equation

$$x + \frac{a^2}{x} = a \coth (h/x)$$

has no real root whenever $h > a\sqrt{3}$. For this we trace the curves

$$y = a \coth (h/x) \quad \dots \quad \dots \quad (A_1)$$

and $$y = x + \frac{a^2}{x} . \quad \dots \quad \dots \quad (A_2)$$

The curve (A_1) is asymptotic to $y = (a/h)x$ and meets the y-axis at $y = a$. Moreover, we find that

$$h > a\sqrt{3} \implies (a/h) < (1/\sqrt{3}) < 1 .$$

Also $$h > a\sqrt{3} \implies a \operatorname{cth} (h/x) < a \operatorname{cth} (\sqrt{3}).$$

Moreover, since $\sqrt{3} = 1.73205 \dots \dots$ and $\operatorname{th} (1.73) = 0.93906 \dots$,

we have $$1 > \operatorname{th} (\sqrt{3}) > 0.93906.$$

It then follows that whenever $h > a\sqrt{3}$, the y-coordinate of the point of intersection of the curve (A_2) with the ordinate through the lowest point of (A_2) is less than 1.0648 a. Thus the curve (A_1) will never meet (A_2) as shown in the figure below:-

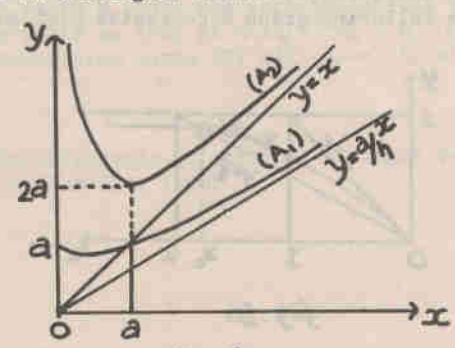

fig. 6

Next towards tracing (C) on the background of (G_1), (G_2) and (G_3) we find that

$$h > a\sqrt{3} \Longrightarrow \sqrt{gh} > \sqrt{3ag} > \sqrt{ag}$$

and, therefore, the asymptote $c = \sqrt{gh}$ of (C) cannot be below the point A, the point of intersection of (G_2) and (G_3). This, in fact, is obvious since the lowest point of (C) has been already shown to be above A.

In view of the fact that the curve (C) always remains between the curves (G_1) and (G_2), we may distinguish the three cases according as the point B (intersection point of the lines $c = \sqrt{gh}$ and AM) lies below M (the lowest point of (G_1)), at M or above M i.e. according as (i) $a\sqrt{3} < h < 2a$, (ii) $h = 2a$ and (iii) $h \quad 2a$ respectively. The minimum of (C) lies on the right hand side of the line AM and is between A and B.

We now present below the curve-tracing of (C), (G_1), (G_2) and (G_3) roughly in the above three cases.

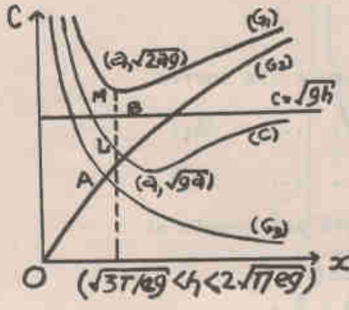

fig. 7

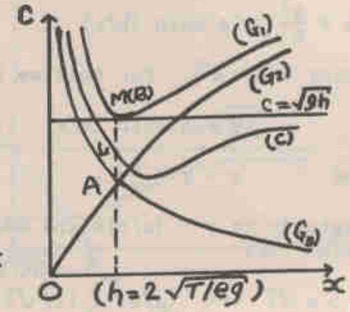

fig. 8

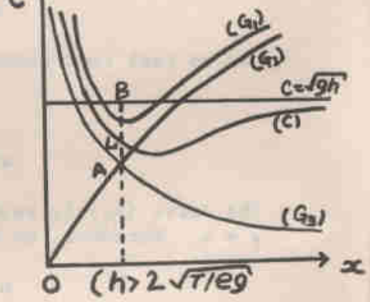

fig. 9.

It is to be noted that the point L (the intersection point of the line AM with the curve (C)) is below the point B (the point of intersection of AM with the line $c = \sqrt{gh}$) in the case (ii) and (iii) as shown in the figures 8 and 9. But in the case (i) the point L may fall between B and M or may even be at B. In fact, we can show that L will be above B for, $x_0 > h/a > \sqrt{3}$ and will coincide with B for $h/a = x_0$, where x_0 denotes the positive real root of the equation, th $x_0 = x/2$. That the equation th $x_0 = x/2$ has a positive real root x_0 such that $\sqrt{3} < x_0 < 2$ is evident from the following graph accompanied with some elementary analysis;

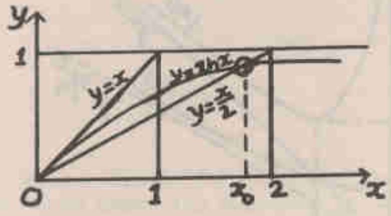

fig. 10

From the above graph we note that $x_0 < 2$. Also $x > \sqrt{3}$ for
$th\sqrt{3} > 0.93906 \ldots > \sqrt{3}/2$ (=0.86602 app.),
showing that $\sqrt{3}$ lies to the left of x_0.

Thus we find that in Fig. 7, the point L is above B so long as $\sqrt{3} < h/a < x_0$ and L coincides with B if $h/a = x_0$. For $h/a > x_0$ the point L will remain always below B. The Fig. 7 has been drawn in this very particular case.

Conclusion

We have found above that the dispersion curve (C) in the (c, λ)-plane has a single minimum if and only if $h > \sqrt{3T/\rho g}$ i.e. the minimum of the curve exists if and only if the depth of the water is greater than $\sqrt{3T/\rho g}$. i.e. the minimum of the curve exists if and only if the depth of the water is greater than $\sqrt{3T/\rho g}$. This depth is usually called the critical depth for the existence of minimum and denoted by h_c. The value of h_c 0.472 cm. app. for water at 20° C, T = 72.8 dynes/cm, $\rho = 0.998$ gm/cm^3 and g = 981 cm/sec^2.

It is now clear from the Fig. 5, that there will be no progressive periodic wave for $c < c_m$, the least value of c in the graph (C). There will be two values of λ for each e in c_m $/\overline{gh}$, both of them coinciding with λ_m for $c = c_m$. Also we shall have only one finite for $c = \sqrt{gh}$, one of the values of becomes infinite and the finite $\lambda < \lambda_m$. This λ is the positive root of the equation

$$x + \frac{a^2}{x} = h \operatorname{cth} (h/x), \qquad (x = \lambda/2\pi \ , \ a^2 = T/\rho g).$$

By graphic method one can show that this equation has one finite root x_m which is less or greater than a according as $h \geqslant 2a$ or $a\sqrt{3} < h < 2a$. The other root x = is in the direction of the line y = x.

In the case $c_m < c < \sqrt{gh}$, the smaller of the roots (say λ_1) of the equation (C) corresponds to waves which are prominently affected by the surface tension. On the other hand, the greater of the roots (say λ_2) of the equation (C) gives waves which are mainly caused by gravity.

It is also interesting to note that the second order infinitesimal wave approximation of progressive periodic waves on the surface of a liquid of constant depth h under the combined effect of gravity and surface tension is valid so long as $h > \sqrt{3T/eg} = h_c$ [1, pp. 659], in this case the dispersion curve (C) will have the minimum point.

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Department of Mathematics,
A.N.S. College, Barh,
Dist- Patna (Bihar)
and
Department of Mathematics,
University of Bihar,
Muzaffarpur (Bihar).

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Life Table Fstimation for Males and Females of Nepal Based on Two Successive Censuses

Ganga Shrestha

Introduction

Life tables are mainly used to measure the level of mortality of the population involved. The most important advantage of this method for measuring mortality is that it does not reflect the effects of the age distribution of an actual population and do not require the adoption of a standard population for acceptable comparisons of levels of mortality in different populations.

This present paper deals with estimating mortality and constructing life tables for males and females of Nepal based on 1961 & 1971 census data. This is an indirect method of estimating mortality and deriving the life table functions by using Logit Model system developed by Brass [2]. Due to the lack of data regarding mortality and vital registration data in Nepal it is not possible to use a direct method for estimating mortality.

In the logit method system the relationship

$$Y(x) = \alpha + \beta Y_s(x)$$

is assumed to be linear.

Here α & β are the constants and $Y_s(x)$ the logits taken from the general standard life table and $Y(x)$ the logits of $(1-l_x)$, l_x being the survivors to exact age x. The survivors to exact age x being then estimated by the equation

$$l_x = \frac{1}{1+e^{2Y(x)}}.$$

On the basis of these l_x values the life table functions were computed.

Application

The childhood mortality was estimated from the data of World Fertility survey maternity history. The childhood mortality for the year 1961-1971 was estimated by taking the average of the estimate of 1961-1966 and 1966-1971.

Intercensal childhood mortality from WFS maternity history, Nepal [6].

	1961-1966	1966-1971	1961-1971
$_2q_0$	.2718	.2240	.2479
$_3q_0$	.2754	.2489	.2621
$_5q_0$	.3090	.2742	.2916

From the estimate of $_5q_o$ i.e. the probability of children at birth dying by age 5, the probability of children serving by age 5 was computed. Then a typical life table characterized by this probability of surviving was selected which in this case is the normal survivorship function of the life table with the radise taken as one. The value of $_5L_5$ (3.49) that is the number of children surviving between age 5 and 10 was taken from Brass one parameter model life table, level 35, carrier & Hoberaft 3 corresponding to the value of l_5 (.7084).

Table No. 1

5 year age distribution of males & females of Nepal from 1961 & 1971 censuses & their 10 year survivorship ratios [4]

Age group	Males			Females		
	1961 Census	1971 Census	Survivorship ratios	1961 Census	1971 Cenus	Survivorship ratios
0-4	661,822	790,598	-	680,467	843,512	-
5-9	688,870	885,801	-	670,960	857,452	-
10-14	564,869	703,023	1.062	498,795	594,192	.873
15-19	408,723	547,493	.795	401,867	499,966	.745
20-24	366,436	466,022	.825	424,904	503,653	1.010
25-29	387,061	456,297	1.116	428,974	473,990	1.179
30-34	336,486	385,696	1.053	372,602	425,705	1.002
35-39	298,423	386,381	.998	287,860	358,407	.835
40-44	222,048	301,998	.898	250,297	307,463	.825
45-49	194,107	245,521	.823	191,820	215,577	.749
50-54	170,861	204,304	.920	186,375	196,530	.785
55-59	113,405	132,983	.685	113,897	124,716	.650
60-64	103,176	138,441	.810	128,907	155,789	.836
65-69	51,443	71,427	.630	55,781	77,336	.626
70 & over	68,303	101,218	.454	83,457	110,493	.412

The last three numbers that is the number of persons in the age groups 60-64, 65-69 & 70 & over of 1961 census were cumulated to get the number of males surviving in the age group 60 & over and this total value was divided by the number of males in the age group 70 & over in the 1971 census to obtain the last survivorship ratio. The same was done in case of females.

The five year survivorship ratios from the ten year survivorship ratios were computed by the relation

$$_5P_x = ({}_{10}P_{x-1} \times {}_{10}P_{x+1})^{\frac{1}{4}}$$

where x stands for any five year age group and x-1 for the one before it.

The $_5L_x$ values for adult age groups were computed by the relation

$$_5L_{x+5} = {}_5L_x \times {}_5P_x$$

The starting value of $_5L_5$ was taken from Brass one parameter model life table.

The survivals at exact age $\overline{x}, l_{\overline{x}}$ that is between age x & x+5 were computed and the logits of $1-l_{\overline{x}}$ denoted by $Y(\overline{x})$ were computed by the relation

$$Y(\overline{x}) = \tfrac{1}{2} \ln \frac{1-l_{\overline{x}}}{l_x}$$

The computation of five year survivorships and their logits are presented in the following table:

Table No. 2

Intercensal survivorships for males & females from 1961 & 1971 censuses & their logits.

Exact age	Males				Females				$Y_s(\overline{x})$
	$_5P_x$	$_5L_x$	$l_{\overline{x}}$	$Y(\overline{x})$	$_5P_x$	$_5L_x$	$l_{\overline{x}}$	$Y(\overline{x})$	
7.5	.9586	3.49	.698	-.419	.898	3.49	.698	-.419	-.571
12.5	.8998	3.345	.669	-.352	.9314	3.135	.627	-.260	-.533
17.5	.9796	3.010	.602	-.207	1.0446	2.920	.584	-.170	-.489
22.5	1.042	2.950	.590	-.182	1.0425	3.050	.610	-.224	-.418
27.5	1.0125	3.075	.615	-.234	.9564	3.180	.636	-.279	-.348
32.5	.9730	3.110	.622	-.249	.9110	3.040	.608	-.219	-.283
37.5	.9272	3.025	.605	-.213	.8866	2.770	.554	-.108	-.216
42.5	.9328	2.805	.561	-.123	.8757	2.455	.491	.018	-.146
47.5	.8910	2.620	.524	-.048	.8452	2.150	.430	.141	-.066
52.5	.8631	2.335	.467	..066	.8586	1.820	.364	.279	.028
57.5	.8452	2.015	.403	..196	.8505	1.560	.312	.395	.143
62.5	.7313	1.700	.340	.332	.7126	1.325	.265	.510	.287
67.5 & over		1.245	.249	.552		.945	.189	.728	.470
Average				-.038				.067	.131

The values of α & β were computed by solving the two linear equations, the 1st equation consisting the values of $Y(\overline{x})$ and $Y_s(\overline{x})$ from the childhood age group given in the first row of table No. 2 and the other consisting the averages of $Y(\overline{x})$ and $Y_s(\overline{x})$ in the latter age groups.

The equations for males are as follows:

$$-.038 = \alpha - .131\,\beta$$

$$-.419 = \alpha - .571\,\beta$$

Solving these two we have $\alpha = .075$ & $\beta = .866$ similarly for females, we have

$$.067 = \alpha - .131\beta$$

$$-.419 = \alpha - .571\beta$$

Solving these two, we have $\alpha = .212$ & $\beta = 1.105$.

After computing the values of Y(x) for males & females by the equation.

$Y(x) = \alpha + \beta Y_s(x)$ and then l_x by the equation

$$l_x = \frac{1}{1+e^{2Y(x)}}$$

the life table functions were computed for males and females and are presented in tables 3 & 4.

Table No. 3

Abridged life table for males of Nepal

Age in years	Survivors at exact age x l_x	Proportion dying between age x & x+n $_nq_x$	Number dying between age x & n+n $_nd_x$	Persons living between age x & x+n $_nL_x$	Persons living in this age & subsequent age T_x	Average remaining life time $\overset{o}{e}_x$
0	100000	.20561	20561	85607	4058245	40.58
1	79439	.10717	8513.4	300730	3972638	50.00
5	70926	.02654	1882.3	349925	3671908	51.77
10	69044	.01954	1349.1	341847	3321983	48.11
15	67695	.03340	2261.0	332822	2980136	44.02
20	65434	.04400	2879.0	319972	2647314	40.45
25	62555	.04464	2792.4	305795	2327342	37.20
30	59763	.04602	2750.2	291940	2021547	33.82
35	57013	.05092	2903.1	277807	1729607	30.33
40	54110	.05994	3216.2	262510	1451800	26.83
45	50894	.07312	3721.3	245167	1189290	23.36
50	47173	.09489	4476.2	224675	944123	20.01
55	42697	.12336	5267.1	200317	719448	16.85
60	37430	.17104	6402.0	171145	519131	13.86
65	31028	.22941	7118.1	137345	347986	11.21
70	23910	.32133	7683.0	100342	210641	8.80
75	16227	.44014	7142.1	63280	110299	6.79
80	9085	.58305	5297.0	32182	47019	5.17
85	3788	.73496	2784.0	11980	14837	3.91
90	1004	.86156	865.0	2857	2857	2.84
95.	139	-	-	-	-	-

Table No. 4

Abridged life Table for females of Nepal

Age in years	Survivors at exact age x l_x	Proportion dying between age x & x+n $_nq_x$	Number dying between age x & x+n $_nd_x$	Persons living between age x & x+n $_nL_x$	Persons living in this age and subsequent age T_x	Average remaining life time $\overset{o}{e}_x$
0	100000	.18362	18362	87146	3681681	36.81
1	81638	.12781	10434.1	305684	3594535	44.03
5	71204	.03370	2399.5	350022	3288851	46.18
10	68805	.02567	1766.2	339610	2938829	42.71
15	67039	.04314	2892.0	327965	2599219	38.77
20	64147	.05840	3746.1	311370	2271254	35.40
25	60401	.06025	3639.1	292907	1959884	32.44
30	56762	.06300	3576.0	274870	1666977	29.36
35	53186	.07047	3748.0	256560	1392107	26.17
40	49438	.08288	4097.4	236947	1135547	22.96.
45	45341	.10278	4660.1	215055	898600	19.81
50	40681	.13341	5427.2	189837	683545	16.80
55	35254	.17315	6104.2	161010	493708	14.00
60	29150	.23716	6913.2	128467	332698	11.41
65	22237	.31111	6918.1	93890	204231	9.18
70	15319	.41968	6429.0	60522	110341	7.20
75	8890	.54736	4866.0	32285	49819	5.60
80	4024	.68664	2763.0	13212	17534	4.35
85	1261	.82158	1036.0	3715	4322	3.42
90	225	.92000	207.0	607	607	2.69
95	18	-	-	-	-	-

From the Demographic sample survey 1976 the expectation of life at birth for males is 43.4 & for females is 41.1 each being higher than that estimated in this case. These estimates seem to be quite reasonable since in 1961-1971 the expectation of life should have been smaller than that in 1976, because of some improvements in the medical facilities.

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Statistics Instruction Committee,
Kirtipur Multiple Campus,
Tribhuvan University,
Kirtipur.

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An Invariance Property of the L. A. M. S. T. for Testing the Hypothesis of no Second Order Interaction in a Three-way Contingency Table

B.N. Nagnur

Summary

Based on the general theory of locally asymptotically most stringent test (L.A.M.S.T., Bhat and Nagnur [4], Nagnur [7] proposed a new class of tests for testing the hypothesis of no second order interaction in a three-way contingency table. In this note, an invariance property of this class of tests is established. The tests proposed by Goodman [5] and the tests based on the min $-\chi_1^2$ statistics do not possess the invariance property discussed in this note.

1. L.A.M.S.T. Statistic for Testing a Non-linear Hypothesis

Let $p(x\ ;\ \underset{\sim}{\theta})$ be the probability density function of a random variable X which is known except for the parameters $\underset{\sim}{\theta} = (\theta_1, \theta_2, \ldots, \theta_s)$. Suppose that we are interested in testing the hypothesis $H_o : \xi_i(\underset{\sim}{\theta}) = 0$, $i = 1, \ldots, r\ (r < s)$, where $\xi_i(\underset{\sim}{\theta})$ is a function of $\underset{\sim}{\theta}$. When the functions $\xi_i(\underset{\sim}{\theta})$ are linear in $\underset{\sim}{\theta}$, the hypothesis H_o can be stated as $H_{o1} : H\underset{\sim}{\theta} = 0$, where H is an r x s matrix. Let $L_n(\underset{\sim}{x}, \underset{\sim}{\theta})$ denote the likelihood function of n independent observations on X, where $\underset{\sim}{x} = (x_1, \ldots, x_n)$. Assuming that that the rank of the matrix H is r, the L.A.M.S.T. criterion for testing H_{o1} is given by Bhat and Nagnur [4].

$$(1.1)\qquad \underset{\sim}{Z}_n = \underset{\sim}{Y}_n' \,(H'V_{\underset{\sim}{\theta}}^{-1}H)^{-1}\, \underset{\sim}{Y}_n,$$

where

$$(1.2)\qquad \underset{\sim}{Y}_n = \frac{1}{\sqrt{n}}\, H'V_{\underset{\sim}{\theta}}^{-1}\, \underset{\sim}{\Phi}_{\underset{\sim}{\theta}},$$

$$(1.3)\qquad \Phi_{\theta_i} = \frac{\partial \log L_n(x,\theta)}{\partial\theta_i}, \quad i = 1, \ldots, s$$

$$(1.4)\qquad \underset{\sim}{\Phi}_{\underset{\sim}{\theta}} = (\Phi_{\theta_1}, \ldots, \Phi_{\theta_s})'$$

and

$$(1.5)\qquad \underset{\sim}{V}_{\underset{\sim}{\theta}} = \frac{1}{n}\, E\left(\frac{\partial^2 \log L_n(x, \underset{\sim}{\theta})}{\partial\theta_i\ \partial\theta_j}\right).$$

We assume that $V_{\underset{\sim}{\theta}}$ is positive definite and $p(x;\ \underset{\sim}{\theta})$ satisfies certain regularity conditions. When n is large, the test criterion $\underset{\sim}{Z}_n$ has an asymptotic x^2 distribution with r degree of freedom (d.f) under H_{o1}. The L.A.M.S.T. criterion for testing H_o can be obtained using the derivation similar to that of $\underset{\sim}{Z}_n$. Nagnur [6]. The derivation needs assumptions regarding $\xi_i(\theta)$ which are used by Wald [9] in deriving a test of H_o based on the maximum likelihood estimate of $\underset{\sim}{\theta}$. The L.A.M.S.T. criterion for testing H_o is given by

$$(1.6) \qquad Z_n^* = Y_n^{*'} (H_\theta' V_\theta^{-1} H_\theta)^{-1} Y_n^* ,$$

where

$$Y_n^* = \frac{1}{\sqrt{n}} H_\theta' V_\theta^{-1} \Phi_\theta ,$$

and

$$(1.7) \qquad H_\theta' = \left(\frac{\partial \xi_i(\theta)}{\partial \theta_j} \right) \qquad \begin{array}{l} i = 1,\ldots, r, \\ j = 1,\ldots, n. \end{array}$$

When n is large, the criterion Z_n^* has an asymptotic χ^2 distribution with r d.f. under H_o. Z_n^* may involve θ_i and to carry out the test, we can substitute any root n consistent estimate of θ_i which is consistent with H_o. Nagnur [7] dealt with a hypothesis which in the form of log-linear contrasts in the parameters of a multinomial distribution and obtained the LAMST test criterion Z_n^*.

2. L.A.M.S.T. Statistic as adapted to test the Hypothesis of no Second Order Interaction

2 x 2 x 2 contingency table: Let θ_{ijk} $i,j,k = 1,2$, be the probability that an observation will fall in the i-th row, j-th column and k-th layer of a three-way contingency table. We assume that $\theta_{ijk} > 0$ and $\Sigma\theta_{ijk} = 1$. For a 2 x 2 x 2 contingency table, the hypothesis of no second order interaction is given by

$$H_{10} : \triangle_1 = \triangle_2, \text{ where } \triangle_k = \frac{\theta_{11k}\ \theta_{22k}}{\theta_{12k}\ \theta_{21k}} ,\ k = 1,2.$$

Goodman [5] considered two other forms of H_{10} i.e., $H_{20} : \log \triangle_1 = \log \triangle_2$ and $H_{30} : 1/\triangle_1 = 1/\triangle_2$ and obtained asymptotic test criteria for testing H_{10}, H_{20} and H_{30}. The tests given by Goodman depend on the cell frequencies i.e., the unrestricted maximum likelihood estimate of θ_{ijk}. These tests can also be obtained by using the general method of testing due to Wald [9]. The three test statistics for testing H_{10}, H_{20} and H_{30} are different. Nagnur [7] obtained a L.A.M.S.T. for testing H_{20}. We, now, derive a L.A.M.S.T. for testing the hypothesis H_{40}: $f(\triangle_1) = f(\triangle_2)$, where f is a single valued differentiable function of $\triangle_k$ and show that the test is independent of f.

Let n_{ijk} be the observed frequency of the ijk-th cell and $n = \Sigma n_{ijk}$. The likelihood function of $\{n_{ijk}\}$ is

$$(2.1) \qquad L_n(x, \theta) = \text{const.} \prod_{i,j,k=1}^{2} \theta_{ijk}^{n_{ijk}} ,$$

where $x = (n_{111}, n_{221}, \ldots, n_{222})$, $\theta = (\theta_{111}, \theta_{221}, \ldots, \theta_{222})$.

Substituting for $\theta_{222} = (1 - (\theta_{111} + \theta_{221} + \dots + \theta_{212})$, from (2.1), we have

$$(2.2) \quad \Phi_{\theta_{ijk}} = \frac{n_{ijk}}{\theta_{ijk}} - \frac{n_{222}}{\theta_{222}} \quad ijk = 111, 221, \dots 212$$

and

$$(2.3) \quad V_{\underset{\sim}{\theta}} = \begin{pmatrix} \frac{1}{\theta_{111}} + \frac{1}{\theta_{222}} & \frac{1}{\theta_{222}} & \cdots & \frac{1}{\theta_{222}} \\ \cdot & \cdots & \cdot & \\ \cdot & \cdots & \cdot & \\ \frac{1}{\theta_{222}} & \frac{1}{\theta_{222}} & \cdots & \frac{1}{\theta_{212}} + \frac{1}{\theta_{222}} \end{pmatrix}$$

The inverse of $V_{\underset{\sim}{\theta}}$ is

$$(2.4) \quad V_{\underset{\sim}{\theta}}^{-1} = \begin{pmatrix} \theta_{111}(1-\theta_{111}) & -\theta_{111}\theta_{221} & \cdots & -\theta_{111}\theta_{212} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ -\theta_{212}\theta_{11} & -\theta_{212}\theta_{112} & \cdots & \theta_{212}(1-\theta_{212}) \end{pmatrix}$$

Also, since H_{40}: $h(\theta_{111}, \theta_{221}, \dots, \theta_{212}) = f(\Delta_1) - f(\Delta_2) = 0$, we get

$$(2.5) \quad H_{\theta}' = \left(\frac{f_1\Delta_1}{\theta_{111}} - \frac{f_2\Delta_2}{\theta_{222}}\right), \left(\frac{f_1\Delta_1}{\theta_{221}} - \frac{f_2\Delta_2}{\theta_{222}}\right), \dots \left(\frac{-f_2\Delta_2}{\theta_{212}}\right)$$

where $f_k = \frac{\partial f}{\partial \Delta_k}$, $k = 1,2$.

Using (2.2), (2.4) and (2.5), the test criterion Z_n^* reduces to

$$(2.6) \quad Z_n^* = \frac{(f_1 \Delta_1 g_1 - f_2 \Delta_2 g_2)^2}{n\,(f_1 \Delta_1)^2 V_1 + (f_2 \Delta_2)^2 V_2}$$

where

$$(2.7) \qquad g_k = \frac{n_{11k}}{\theta_{11k}} + \frac{n_{22k}}{\theta_{22k}} - \frac{n_{12k}}{\theta_{12k}} - \frac{n_{21k}}{\theta_{21k}}$$

and

$$(2.8) \qquad v_k = \frac{1}{\theta_{11k}} + \frac{1}{\theta_{12k}} + \frac{1}{\theta_{12k}} + \frac{1}{\theta_{21k}}$$

When H_{40} is true, we have, $f_1 = f_2$ and $\Delta_1 = \Delta_2$ and hence the expression in (2-6) reduces to

$$(2.9) \qquad Z_n^* = \frac{(g_1 - g_2)^2}{n(v_1 + v_2)}$$

which is independent of f. It is interesting to note that the test criterion (2.9) is same as the L.A.M.S.T. criterion for testing H_{20}.

It can be verified that the test criterion (2.9) also satisfies the requirements of symmetry, i.e., invariance under relabelling of the categories.

3. Equivalence of the Min $-\chi_1^2$ Statistic and the Test Statistic based on the Unrestricted m.l.e's

Goodman [5] obtained some test criteria, which are based on the unrestricted m.l.e's of the cell probabilities, for testing the hypothesis of no second order interaction in a three-way contingency table. In a 2 x 2 x 2 contingency table, for testing

$$(3.1) \qquad H_{20}: \sum_{ijk} l_{ijk} \log \theta_{ijk} = 0 \quad l_{ijk} = \begin{cases} 1 \text{ for } (ijk) = 111, 221, 122, 212, \\ -1 \text{ for } (ijk) = 121, 211, 112, 222, \end{cases}$$

the statistic given by Goodman is

$$(3.2) \qquad Y^2 = \left(\sum_{ijk} l_{ijk} \log n_{ijk}\right)^2 \Big/ \sum_{ijk} \frac{1}{n_{ijk}} ,$$

where n_{ijk} is the observed cell frequencies. The test statistic Y^2 can be obtained by using the general method of testing due to Wald [9].

Another test statistic which can be used for testing H_{20} is the min $-\chi_1^2$ statistic which is given by

$$(33) \qquad \chi_1^2 = \sum_{ijk} (n_{ijk} - n\,\hat{\theta}_{ijk})^2 / n_{ijk},$$

where $\hat{\theta}_{ijk}$'s are the estimates of θ_{ijk}'s which minimise
$\sum_{ijk} (n_{ijk} - n\,\theta_{ijk})^2/n_{ijk}$ subject to H_{20}. The statistic X_1^2 has an asymptotic X^2 distribution with one degree of freedom, under H_{20}. For the purpose of computing θ_{ijk}, we can use linearised form of H_{20} Neyman [8], which is given by

$$(3.4) \qquad H_{20}' : \sum_{ijk} l_{ijk} \log n_{ijk} + \sum_{ijk} l_{ijk} \frac{n\theta_{ijk}}{n_{ijk}} = 0$$

The min $-\chi_1^2$ estimates $\hat{\theta}_{ijk}$ are obtained by minimising

$$(3.5) \qquad = \sum_{ijk} (n_{ijk} - n\theta_{ijk})^2 /n_{ijk} + 2\lambda \sum_{ijk} l_{ijk} \log n_{ijk}$$

$$+ \sum_{ijk} l_{ijk} \frac{n\theta_{ijk}}{n_{ijk}} + 2n\mu\,(\sum_{ijk} \theta_{ijk} - 1),$$

where 2λ and $2n\mu$ are the Lagrangian multipliers. The estimating equations for $\hat{\theta}_{ijk}$ are given by

$$- (n_{ijk} - n\hat{\theta}_{ijk}) /n_{ijk} + l_{ijk} /n_{ijk} + \mu = 0,$$

i.e.,

$$(3.6) \; - (n_{ijk} - n\hat{\theta}_{ijk}) + l_{ijk} + \mu\, n_{ijk} = 0 \quad \text{for all } (ijk)$$

Since $\sum_{ijk} l_{ijk} = 0$, summing the equations in (3.6) for all (ijk), we get, $\mu = 0$. Therefore, $\hat{\theta}_{ijk}$ must satisfy the equation

$$(3.7) \qquad n_{ijk} - n\hat{\theta}_{ijk} = \lambda . l_{ijk}$$

From (3.3) and (3.7), we have

$$(3.8) \qquad X_1^2 = \lambda^2 \sum_{ijk} 1/n_{ijk}$$

Also, from (3.4) and (3.7), we have

$$(3.9) \qquad - \sum_{ijk} l_{ijk} \frac{n\theta_{ijk}}{n_{ijk}} = \sum_{ijk} l_{ijk} \log n_{ijk} = \lambda \sum_{ijk} 1/n_{ijk}$$

From (3.2) and (3.9), we have

$$Y^2 = \lambda^2 \sum_{ijk} 1/n_{ijk}$$

which is equal to χ_1^2 .

Similarly we can establish the equivalence of the two test statistics for testing H_{01} and H_{03}. Further we can extend the results to other three-way contingency tables. A general result concerned with the equality of the Wald test and the min $- \chi_1^2$ test statistics is established for a two-way contingency table by Bhapkar [3].

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Karnatak University,
Dharwar, India.

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On The Teaching of Mathematical Symbols

George F. Feeman

During my tour of duty as a mathematics specialist in Nepal from 1975 to 1977, Dr. Prem Kasaju, Omkar Nath Pant, Achyut Man Pradhan, Sushil Kumar Shrestha, and I conducted a research study on the use of Devnagri and Hindu-Arabic numerals in the Lower Secondary Schools. The results of this study were reported in [3]. In general, it was found that students experienced difficulties in utilizing the two sets of symbols which are taught in their mathematics classes. Some of these difficulties resulted from the likenesses of certain symbols in the two sets. Others probably resulted from inadequate emphasis and reinforcement in the teaching of them.

These symbolic or notational difficulties are not new. They have existed in mathematics for many centuries, and no doubt will continue to exist. In our present day, with the prevalence of computers, new types of difficulties have arisen. Symbols are part of mathematics. They help to construct its language in the written, representational form. For this reason, mathematics is sometimes referred to as a symbolic system. It is important for teachers of mathematics to recognize the problems which exist and, in their teaching, to combat them in a satisfactory way. The purpose of this article is to discuss several approaches which a teacher might use to strengthen his or her efforts in this direction.

Let me begin by citing some special cases, so that the reader knows precisely what I am talking about. First, every teacher of mathematics has encountered difficulties in the teaching of place value and lack of understanding on the part of students.

> What is the meaning of the "5" in "4,257?"
> What is the name for 32,57,649?

And so on. This is one type of problem. In algebra classes, it is the perennial problem of shifting from numbers to variables and algebraic expressions - the meaning of x, the confusion of x with 0x (nothing is in front of x, so it must be 0!), the confusion of $2x^2$ with $(2x)^2$, and the value of 5x for x = 7 being 57, rather than its real value of 5 times 7 or 35. These are typical difficulties which arise.

Second, there are disagreements over the use of symbols - when to introduce them, how to introduce them, and how much usage to permit for pedagogical soundness. For example, some teachers object to the use of the symbols for greater than (>) and less than (<) in the primary classes. They feel that the symbols are confusing and meaningless, while the ideas themselves are intuitively clear. Whatever problems arise do so because of the symbols, not because of the concepts. If symbols must be used why not use GT and LT? The same arguments apply

for other symbols. The trend to the "new mathematics" in the 1960's brought with it an excessive use of symbols. Now we are experiencing a retrenchment with a better balance in presentation and use of symbols.

Third, there is confusion over the use of multiple forms in the minds of many teachers and students. Again it was the "new mathematics" which brought about this development. Horizontal forms, such as 37 + 24 = ?, with their sentence-like structure became much more prevalent. As a result, arguments arose on the respective teaching of these versus the more traditional vertical forms, as typified in

$$\begin{array}{r} 27 \\ +\underline{24} \end{array}$$

Are the bar "__" in the second and the "=" in the first the same or not? How should these forms be taught conceptually? Since children have no prior knowledge of the use of mathematical symbols, it is up to the teachers to decide the answers to these questions. Some teachers believe distinctions should be emphasized, others feel that it is not important.

Fourth, the computer has brought with it new problems and new needs for explicit coding. For example, what does 12+12 x 24 ÷ 6 equal? By the BODMAS rule, which is commonly taught in the Nepalese curriculum, the answer is 60. However, some calculators are programmed to do the operations in the order in which they appear. In this case the answer is 96. What are we to teach the children? Should we continue to teach a BODMAS type rule, or should we discuss different alternatives? These questions are difficult to answer. There may be no universal answers since practices change. As with other things, mathematical symbols evolve over the years. What is an answer in one year may not be the same the next year.

These examples show the nature of the problems. Let us discuss several things teachers can do to teach mathematical symbolism more effectively.

Symbols, forms, and meanings are the operative parameters. As any reading or language teacher knows, all three must be dealt with for the children to gain proper understanding. In mathematics, the situation is compounded because teachers and children alike must contend with multiple forms, multiple symbols, and multiple meanings. Positions of symbols need not be fixed because of commutative and associative laws which prevail. Teachers must continually discuss and emphasize the ideas and concepts, the objects or models which represent them, the words and vocabulary which name them and which ease oral and written communication, and the symbols which help to shorten presentations and reveal structure.

What are symbols? Three characteristics are usually given for a symbol to be valid. They are:

(1) A symbol must be representative. It is an event which stands for another event. For example, "let x be ... " is a common occurrence in algebra.

(2) A symbol must be freely created. That is, the choice is somewhat arbitrary. There is no reason why any symbol need stand for any particular event. For example, $x + 2 = 5$ and $t + 2 = 5$ are the same sentence. The use of the x or the t is arbitrary. Any letter will do.

(3) A symbol must be transmitted by culture. That is, symbols are taught and learned and then passed down.

As teachers introduce symbols, they must convey to the students the idea that symbols are vehicles for the conception of objects. They are not the objects. To perceive a thing is not the same thing as being aware of its presence. For example, how often do students "see" symbols, but not "perceive" them? The symbols are seen but it is as if they are not there in any substantive way, as perhaps specks of dirt on the page. Thus the perception of a symbol is a great deal more than the seeing of it. As teachers of mathematics, we must assist students in developing the ability to perceive symbols, to understand them, and to have feeling for them.

What is a symbolic system? Usually four characteristics are given. They are:

(1) Symbols must have the ability to be applied to a variety of situations. For example, the symbol "2" stands for "twoness" and represents any set of two objects. It can be applied to a variety of situations.

(2) Mathematical symbols do not stand alone but are organized into systems which are governed by explicit laws and rules.

(3) These rules provide this particular system with the ability to generate increasingly more general and more powerful statements. They also define statements which are permissible within a system. (Frequently it is this accumulation of generality and power, along with greater precision, which confound our students).

In short, the first three conditions say that there is a vocabulary and a set of combinatorial rules which allow one to operate on the vocabulary elements.

(4) There is the property of multiple expression. This property makes it possible to construct a dictionary and to translate from one form to another as well as from one symbolic system to another, once the rules of translation are known.

If one views mathematics as a symbolic system, along with language, and other disciplines, then the discussions in the various disciplines are not distinct. All talk about the same thing - vocabulary, syntax, and the property of multiple expression. These things are transmitted by culture; they are taught and learned. Clearly the issues involved are both cultural and pedagogical. The specific symbolic activities are affected by the educational system and by individual learning. No child is ever born with a knowledge of his/her society's particular symbols. That must always be kept in mind.

As teachers we cannot ignore these facts. The difficulties will not go away by wishing. Teachers of mathematics must regard themselves as teachers of symbolic systems, as teachers of language. If they fail to do this, the difficulties will simply continue, unresolved. If they do it, then the level of mathematical literacy can be raised substantially.

Let us look at the practical side. To say that teachers must do a certain thing is one thing. To deal with the practicalities is another. There are no easy answers. Awareness of the problem must come first. Repetition and reinforcement are vital.

Some examples of strategies which can be used to help children read mathematics include vocabulary exercises, given singly or in matching form,classification exercises to show relationships among such things as geometric notions, algebraic notions, arithmetic notions, and combinations of these, as well as exercises on definitions. Some of these ideas have already been incorporated in booklets prepared by the mathematics specialists at the CDC-HMG for use in Primary and Lower Secondary level workshops. They should be incorporated into the curriculum materials themselves to get wider exposure.

To determine the magnitude of the problem locally, one might engage in a research project of the following sort. Make a list of mathematical symbols which are commonly taught in the curriculum through Class 7. Classify them as to whether students are to have mastered them or simply been exposed to them. Choose only those in the "mastery" category. Now test the students of Class 7 to see what percent actually have mastered them. Which symbols are known by 90% of the students, which by 80-90%, which by 70-80%, and so on? Which symbols are known to less than 50% of the students? The results will probably be surprising. The magnitude of the problem will become apparent. Surely this has some bearing on the performance of students on Lower Secondary examinations.

Some American educators did this type of experiment in some American schools in grades 7 and 8. Their results are reported in [2]. Here is a summary of them.

90-100% familiarity: +, −, x, \$, ÷, %, ¢

80-90% familiarity : $\overline{)\quad}$, V, II, "−" as in $\frac{9}{3}$, "$\_$" as in $+\frac{8}{\underline{2}}$, 2 as in 4^2

70-80% familiarity: $^{+}$ as in $^{+}5$, . as in 2.46, x, ° as in 90°, =, >

60-70% familiarity: $^{-}$ as in $^{-}4$, , " for inches, $\neq$, ^{3} as in x^3

Symbols such as

$$\triangle\ , \{\ \}, (\ ), \leftrightarrow, \cap\ . \not>, \emptyset, \cup, \pi, \rightarrow, \parallel, \not<, \sqrt{\ },$$

$$[\ ], \geq, \approx, |\ |, \subset, \cong, \in\ ; \notin, \therefore, \equiv$$

and others were known by less than 50% of the students tested. Notice that all geometry symbols were in this category.

What would the situation be among Nepalese children? Of course, the symbols would be different since Rs. and p. would replace \$ and ¢, respectively, while the Devnagri numerals would replace the Hindu-Arabic numerals. But would the results differ? Certainly this would be useful information for any teacher to have. Once it is available, appropriate corrective measures can be taken.

Studies such as this show that teachers must work harder to teach symbols, their meanings, and their relationships. By viewing mathematics as the symbolic system which it is and by understanding the nature and characteristics of such a system, teachers can help themselves to do a better job. Many of the problems will be placed in a proper context. Many of them can then be solved. For example, story or word problems have caused difficulties for all teachers. In the context we are discussing they can then be viewed as problems in which a student must transfer from one system to another. The recognition of this alone may help to lead to solutions of the pedagogical issues involved.

Next, we should mention the need for a consistent vocabulary of mathematical (and scientific) terminology. With the required use of Nepali language in the classrooms came to need for vocabulary. Previously, English terms were most often used. The editors of this journal have chosen to use it as a vehicle for the development of such a vocabulary. That is a commendable thing indeed. As it is completed and put into practice in the preparation of curriculum materials, the entire mathematics programme should be enhanced.

Finally, let us comment on the history of mathematical symbolism. We have remarked several times that children have no previous knowledge of symbols and their uses. These are things which are acquired. Knowledge of the history of mathematical notation can be invaluable to the teaching of the subject of mathematics. For this, one should consult the definitive and excellent works of Florian Cajori cited in [1]. His study constitutes a mirror of past and present conditions in mathematics which bear on our present notational problems. Knowledge of the successes and failures of the past can contribute to the solution of the problems of today.

Think how vital a classroom situation would become were a teacher familiar with

- the struggles involved in the evolution of the numerals to obtain the forms we now use, and how the computer threatens these once again.

- the struggles involved in the development of the decimal system of numeration, through the Babylonians, Phoenicians, Hebrews, Greeks, Arabs, Romans, Hindus, and others, and now the present trend toward internationalization of the metric system.

- the development of the operational signs + and - and their ultimate finalization in the latter part of the 15th century A.D. in Germany, not too many years ago!

- the struggles for the development of other symbols such as the "times" sign (x) in the 17th century, zero, and others at all levels of difficulty.

Knowledge of these things and other of an historical nature could breathe life into a classroom and would go a long way toward solving some of the problems we have discussed. The greater one's perspective, the better it is to achieve solutions to problems. What is needed are a series of articles, perhaps joint ones by historians and mathematicians, which would discuss the history of mathematics and symbols in Nepal.

Some items are quite interesting. For example, Cajori mentions that in 1951, there were nine different uses for the decimal point (°). In particular, American and European uses are reversed. To an American 2.5 means 2 x 5 = 10, while 2.5 means $2\frac{5}{10}$. To European, 2.5 means 2 x 5 = 10, and 2.5 means $2\frac{5}{10}$. Similarly, in America, "into" connotes division, while in Nepal it refers to multiplication. Cajori also points out that over time mathematicians have been notably inefficient in development of notation and symbols. First, they can't agree. Second, if they do agree, they don't adhere to the agreement.

Easy or quick solutions to symbolic and notational problems do not exist. The approaches we've discussed may help, but not completely. Nevertheless, they are worth a try. The more knowledgeable the teacher is philosophically, historically, and practically, the more easily and effectively can the students be taught. This is what we must all strive for if the level of mathematical literacy is to be raised.

References

[1] Cajori, Florian, A History of Mathematical Notations, Volumes 1 and 2, Open Court Publishing Company, 1951.

[2] Kane, Robert, Helping Children Read Mathematics, American Book Company, 1974.

[3] Kasaju, Prem, et al, "Use of Devnagri and Hindu-Arabic Numerals in the Lower Secondary Schools," Education and Development, Centre for Educational Research, Innovation and Development, Kathmandu, Nepal, 1977, pp. 15-26.

Oakland University
Rochester, Michigan, U.S.A.

GLOSSARY OF MATHEMATICAL TERMS
(Proposed)
(G)

Galactic आकाश गङ्गीय,तारामण्डीय

Galaxy आकाश गङ्गा, नक्षत्र मण्डल

Gallon ग्यालन

Galois field ग्याल्वा क्षेत्र

Galois group ग्याल्वा समूह

Game खेल

Gamma (√) गामा

Gauche विषमतली, कर्कश

Gauge मापन गर्नु

Gauss गौस

G.C.M. महत्तम समापवर्तक

General सामान्य

General equation सामान्य समीकरण

Generalisation सामान्यीकरण

Generalise सामान्यीकरण गर्नु

Generality सामान्यता

Generate पैदा गर्नु, निकाल्नु, उत्पादन गर्नु

Generating line प्रजनक रेखा, उत्पादक रेखा

Generating function प्रजनक फलन

Generator प्रजनक, उत्पादक

Geodesic भूगणितीय, भूवक्रगणितीय

Geodesy भूगणित, भूवक्रगणित

Geometry ज्यामिति, रेखा गणित

Glide चिसर्पण, चिप्लनु

Glisette विसर्पिका

Google ($१०^{१००}$) दश-शतगुणित

Grade क्रम, स्तर, श्रेणी

Gradient ढाल, प्रवणता

Graduation अंशाङ्कन

Gramme ग्राम

Graph ग्राफ, लेखा चित्र

Graphic लेखा चित्रिय

Graphical लेखा चित्रित

Graphically लेखा चित्रीय तवरले

Gravitation गुरुत्वाकर्षण

Gravitational गुरुत्वाकर्षी, गुरुत्वीय

Gravity गुरुत्व

Gross कुल, जम्मा, बाह्र दर्जन

Group समूह

Grouping समूहिकरण

Gyration विघूर्णन, परिक्रमण

Gyroscope घूर्णदर्शी

Gyrostat घूर्णस्थापी

(H)

half आधा

harmonic सुसंवादी, हरात्मक

harmonics सम्वादिनी

यो शब्दार्थ त्रि० वि०, कीर्तिपुर बहुमुखी क्याम्पस, गणित तथा नेपाली शिक्षण समितिले संयुक्त रूपमा तयार गरिएको हो ।

Havrsine अर्धशरज्या

H.C.F. महत्तम समापवर्तक

Height उच्चता

Helicoid सर्पिल कुण्डली, सर्पिलज

Heliocentric सूर्यकेन्द्री

Helix कुण्डलिनी

Heptagen सप्तभुज

hessian हैसियन

Heterogeneous विजातीय, विसंङ्गत

Heuristic स्वत:शोध

hexagen षड्भुज, षट्कोण

Hexagonal षट्भुजाकार, षट्कोणीय

Histogram आयत चित्र

Historiogram कालिक चित्र

Hodograph होडो ग्राफ, वेगालेख

hollow खोखला

Holomorphic सर्वत्र अवकलनीय, सर्वसंरचनात्मक

holonomons उभय स्वतन्त्र, सर्वस्वतन्त्र

Homocyclic समचक्रिय

homogeneous सजातीय, सम, समघातीय

Homographic एकस्वरूप, एकालेखी

homograph एकरूपिता

homologous सजातीय, सधर्मी, समधर्मी

homomorphic समरूपी, सरूप

Homomorphism समरूपता, सारूप्य

Homothetic समस्थितिक, समस्थितीय

Horizon क्षितिज

Horizontal क्षैतिज, क्षितिजीय

Horograph समयालेख

Hundredweight हण्ड्रेडवेट

Hydraulic द्रव चालित, जल चालित

Hydrodynamic द्रव गतिक

Hydrodynamics द्रव-गति-विज्ञान

Hydrokinetic द्रवगतिकीय

Hydromechanics द्रव यान्त्रिकी

Hydrometer द्रव-घनत्व मापक

Hydrostatic जलस्थैतिक

Hydrostatics द्रव-स्थिति-विज्ञान

hyperbola अतिपरवलय

hyperbolic अतिपरवलयिक

Hyperboloid अतिपरवलयज

hypercomplex अतिसम्मिश्र

Hypergeometric अतिगुणोत्तर

Hyperharmonic अतिसुसंवादी

hyperplane अतिसमतल

hyperspace परावकाश

hypersphere गोलोत्तर

Hypocycloid अन्तश्चक्रज

hypotenuse कर्ण

Hypothesis परिकल्पना, प्राक्कल्पना

hypothetical परिकल्पनात्मक, प्राक्कल्पित

(I)

Idempotent समदाम

Identical समरूप, सदृश

Identically सर्व समत

Identity समरूपता, सादृश्य

Illogical तर्क विरूद्ध, असङ्गत

Image बिम्ब, प्रतिबिम्ब

Imaginary कल्पित, काल्पनिक

Imbedding अन्तर्भूत, अन्तर्गामी

Immediate तत्काल, अव्यवहित

Impact संघट्टन

Impinge ठक्कर खानु

Implicit अंतर्निहित, अन्त: स्थित

Impressed आरोपित

Improper अनुचित

Impulse आवेग

Impulsive आवेगी

Inaccurate गलत, अशुद्ध

Incentre अन्त; केन्द्र

Inch इन्च

Inclined नत, झुकेको

Inclination झुकाव, नति

Included समाविष्ट, समावेशित

Incommensurable असम्मेय, अतुलनीय

Incommensurability असम्मेयता, अतुल्यता

Incomplete अपूर्ण, असंपूर्ण

Incompressible असंपीड्य

Inconsistent सामन्जस्यहिन, असंगत

Increasing बढ्ने, वर्धमान

Increament संवृद्धि, वृद्धि

Indefinite अनिश्चित

Independent स्वतन्त्र, स्वाधीन

Indeterminate अनिर्धारित, अनिर्दिष्ट

Index घाताङ्क

Indicator संङ्केतक

Indirect अप्रत्यक्ष, परोक्ष

Induction आगमन

Inelastic अस्थितिस्थापक, अप्रत्यास्थित

Inequality असमता, वैषम्य

Inertia अवस्थितित्व, यथास्थितित्व

Inextensible अविस्तार्य, अवितान्य

Inference अनुमिति

Infinite अमित, अनन्त

Infinitesimal अत्यणु, अनन्तसूक्ष्म

Infinity अनन्तता, अमितता

Inflation स्फीति, स्फीतिकरण

Initial आदि, आरंभिक

Inner भित्री, अन्तर

Inseparable अविभाज्य, अपृथक्करणीय

Instability अस्थिरता

Instalment किस्ता

Instance दृष्टान्त

Instantaneous तत्क्षणिक, तात्कालिक
Integer पूर्णसङ्ख्या, पूर्णाङ्क, सकलाङ्क
Integrability समाकलनीयता
Integrable समाकलनीय
Integral पूर्णसङ्ख्या, अविकल
Integrand समाकल गरिइने, समाकल्य
Integrate समाकलन गर्नु
Integrating समाकलन, समाकलन गर्ने
Integration समाकलन
Integrators समाकलकहरू
Interest ब्याज
Interior अन्तरङ्ग, अन्तरांश
Internal आन्तरिक
Internally आन्तरिकता पूर्वक
Interpolation निवेशन, अन्तर्वेशन
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Interated पुनरुक्त
Interation पुनरुक्ति
Intersecting प्रतिच्छेदी
Intersection प्रतिच्छेद, कटान, टुक्रा
Interval अन्तराल, अन्तर
Into गुणा ( ), गुणन
Intrinsic निजी
Intuitive अन्तस्फुर्त, स्वयंस्फूर्त, स्वस्फूर्त
Invariable अचर, परिवर्त्य
Invariant अचलराशिक, निश्चर
Inverse प्रतिलोम, विलोम
Inversely प्रतिलोमत; उल्टो तवरले
Inversion प्रतिलोमीकरण
Invertible प्रतिलोम्य
Invisible अदृश्य
Involute प्रतिकेन्द्रज
Involution घातकरण, घातक्रिया
Irrational असंमत, तर्क विरुद्ध
Irreducible अलघुकरणीय
Irresolvable असंहनीय, असमाधेय
Irreversible अप्रत्यावर्ती, अविचाल्य
Irrotational अघूर्णनीय, अनावर्त्य
Isocline समनत रेखा
Isogonal तुल्य कोणीक रेखा, सम कोणिक रेखा
Isomorphic तुल्य रुपी, समरुपी
Isomorphism समरुपता, तुल्यरुपता
Isosceles समद्विबाहु, समद्विभुज
Isotropy समचिन्हता, समदिक्ता

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