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2. Stokes, G.G., On the effect of the internal friction on the motion of pendulums, Trans. Camb. Phil. Soc., 9 (1851), 8-106.

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Submanifolds of H-Structure Manifold

Ram Nivas

1. Preliminaries

Let M^n be an n-dimensional differentiable manifold of class C^∞. Suppose there exist on M^n a (1, 1) tensor field F and a Riemannian metric G satisfying

(1.1) $$F^2 = a^2 I$$

where 'a' is any complex number and I denotes unit tensor field and

(1.2) $$G(F\lambda, F\mu) + a^2 G(\lambda, \mu) = 0$$

for arbitrary vector fields λ, μ on M^n.

Let 'F be tensor field of type (0,2) on M^n given by

(1.3) $${}'F(\lambda, \mu) = G(F\lambda, \mu).$$

It can be easily shown that $'F(\lambda, \mu)$ is Skew-symmetric [1].

The manifold M^n satisfying (1.1) and (1.2) is said to possess the Hermite structure or briefly H-structure ([1], [2]).

An m-dimensional differentiable manifold V is said to possess (a, ϵ) metric structure if there exists a tensor field f of type (1,1) a vector field P, a 1-form p and a metric tensor g satisfying

(1.4) $$f^2 = a^2 I + \epsilon\, p \otimes P\ ,$$

(1.5) $$F(fX) = \theta\, p(X)$$

and

(1.6) $$g(fX, fY) + a^2 g(X,Y) + \epsilon\, p(X)p(Y) = 0$$

where ϵ is a real constant, X,Y arbitrary vector fields and θ, a function on V.

2. Submanifolds

Let M^{n-q} be submanifold of M^n of co-dimensions =, = n. Suppose there exist on M^{n-q}, a (1,1) tensor field $\underset{q}{F}$, q 1-forms $\overset{x}{p}$, q vector fields $\underset{x}{P}$, scalar fields θ^x_y x, y = 1,2,..., q and a metric tensor $\underset{q}{G}$ satisfying

$$(2.1) \qquad (\underset{q}{F})^2 = a^2 I + \epsilon \overset{x}{p} \otimes \underset{x}{P},$$

$$(2.2) \qquad \overset{y}{p}(\underset{q}{F}) = \theta_x^y \overset{x}{p}$$

and

$$(2.3) \qquad G(F \overset{*}{X}, F \overset{*}{Y}) + a^2 G(\overset{*}{X}, \overset{*}{Y}) + \epsilon \overset{x}{p}(\overset{*}{X}) \overset{x}{p}(\overset{*}{Y}) = 0$$

where ϵ is a real constant and $\overset{*}{X}$, $\overset{*}{Y}$ arbitrary vector fields on M^{n-q}.

Thus M^{n-q} satisfying (2.1), (2.2) and (2.3) will be called as "Generalised (a, ϵ) metric manifold of order q."

Since M^{n-q} is a submanifold of M^n, there exists an immersion map $i^* : m^{n-q} \longrightarrow M^n$ and if B^* denote differential of i^*, a vector field $\overset{*}{X}$ in the tangent space of M^{n-q} corresponds to a vector field $\overset{*}{B} \overset{*}{X}$ in that of M^n. We can put [4]

$$(2.4) \qquad F \overset{*}{B} \overset{*}{X} = \overset{*}{B} \underset{q}{F} \overset{*}{X} + \sqrt{\epsilon}\, \overset{x}{p}(\overset{*}{X}) \underset{x}{N}$$

and

$$(2.5) \qquad F \underset{x}{N} = \sqrt{\epsilon}\, \overset{*}{B} \underset{x}{P} - \theta_x^y \underset{y}{N}$$

where $\overset{x}{p}$, $\underset{x}{P}$ are respectively q 1-forms and q vector fields and $\underset{x}{N}$ being q mutually orthogonal unit normals in M^{n-q}.

Premultiplying (2.4) by F and making use of the equations (1.1), (2.4) and (2.5), we get

$$a^2 \overset{*}{B} \overset{*}{X} = \overset{*}{B}(\underset{q}{F})^2 \overset{*}{X} + \sqrt{\epsilon}\, \overset{x}{p}(\underset{q}{F} \overset{*}{X}) \underset{x}{N} + \sqrt{\epsilon}\, \overset{x}{p}(\overset{*}{X}) \left\{ -\sqrt{\epsilon}\, \overset{*}{B} \underset{x}{P} - \theta_x^y \underset{y}{N} \right\}$$

or

$$a^2 \overset{*}{B} \overset{*}{X} = B(\underset{q}{F})^2 \overset{*}{X} + \sqrt{\epsilon}\, \overset{y}{p}(\underset{q}{F} \overset{*}{X}) \underset{y}{N} - \epsilon \overset{*}{B} \overset{x}{p}(\overset{*}{X}) \underset{x}{P} - \sqrt{\epsilon}\, \theta_x^y \underset{y}{N} \overset{x}{P}(\overset{*}{X}).$$

Thus we obtain, on comparing tangential and normal vectors

$$(2.6) \qquad (\underset{q}{F})^2 = a^2 I + \epsilon \overset{x}{p} \otimes \underset{x}{P}$$

and

$$(2.7) \qquad \overset{y}{\underset{q}{p}}(F) = \theta^{y}_{x}\,\overset{x}{p} \, .$$

In view of the equation (1.2) we can write

$$G(F\overset{*}{B}\overset{*}{X}, F\overset{*}{B}\overset{*}{Y}) + a^{2}G(\overset{*}{B}\overset{*}{X}, \overset{*}{B}\overset{*}{Y}) = 0 \, .$$

In view of (2.4), the above equation takes the form

$$(2.8) \qquad G(\overset{*}{B}\underset{q}{F}\overset{*}{X} + \sqrt{\epsilon}\,\overset{x}{p}(\overset{*}{X})\underset{x}{N}, \overset{*}{B}\underset{q}{F}\overset{*}{Y} + \sqrt{\epsilon}\,\overset{x}{p}(\overset{*}{Y})\underset{x}{N}) + a^{2}G(\overset{*}{B}\overset{*}{X}, \overset{*}{B}\overset{*}{Y}) = 0 \, .$$

If $\underset{q}{G}$ be induced metric on M^{n-q}, we have [4]

$$\text{(i)} \quad G(\overset{*}{B}\overset{*}{X}, \overset{*}{B}\overset{*}{Y}) = \underset{q}{G}(\overset{*}{X}, \overset{*}{Y}) \, ,$$

$$\text{(ii)} \quad G(\underset{x}{N}, \underset{y}{N}) = \delta^{x}_{y}$$

$$\text{(iii)} \quad G(\overset{*}{B}\overset{*}{X}, \underset{x}{N}) = 0.$$

Thus (2.8) takes the form

$$(2.10) \qquad \underset{q}{G}(\underset{q}{F}\overset{*}{X}, \underset{q}{F}\overset{*}{Y}) + a^{2}\underset{q}{G}(\overset{*}{X}, \overset{*}{Y}) + \epsilon\,\overset{x}{p}(\overset{*}{X})\overset{x}{p}(\overset{*}{Y}) = 0 \, .$$

By virtue of equations (2.6), (2.7) and (2.10), we have the theorem:

Theorem (2.1). If M^{n} be a manifold admitting H-structure, its submanifold M^{n-q} of codimensions q is generalised (a, ϵ) metric manifold of order q.

3. Hypersurfaces

Results on hypersurfaces can be obtained as particular case of section 2.

Let M^{n-1} be hypersurface of M^{n} and let B denote differential of immersion $i: M^{n-1} \rightarrow M^{n}$. Hence a vector field X in the tangent space of M^{n-1} corresponds to a vector field BX in that of M^{n}. Operating F to BX and unit normal N to M^{n-1}, we obtain FBX and FN in the form [3]

$$(3.1) \qquad FBX = BfX + \sqrt{\epsilon}\; p(X)N ,$$

$$(3.2) \qquad FN = -\sqrt{\epsilon}\; BP - \theta N ,$$

where f, p, P and θ are respectively a (1, 1) tensor field, a 1-form, a vector field and a function on M^{n-1}.

Operating F both sides of (3.1) and making use of the equations (1.1), (3.1) and (3.2), we get

$$a^2BX = Bf^2X + \sqrt{\epsilon}\; p(fX)N + \sqrt{\epsilon}\; p(X) \left\{ -\sqrt{\epsilon}\; BP - \theta N \right\}$$

which, on comparing tangential and normal vectors, yields

$$(3.3) \qquad f^2 = a^2 I + \epsilon\, p \otimes P$$

and

$$(3.4) \qquad p(fX) = \theta\, p(X).$$

If g be induced metric on the hypersurface M^{n-1}, we have

$$(3.5) \qquad G(BX, BY) = g(X, Y)$$

and

$$(3.6) \qquad G(BX, N) = 0 ,$$

for arbitrary vector fields X, Y on M^{n-1}.

In view of equation (1.2), we can write

$$G(FBX, FBY) + a^2 G(BX, BY) = 0$$

which, in view of (3.1), (3.5) and (3.6) takes the form

$$(3.7) \qquad g(fX, fY) + a^2 g(X, Y) + \epsilon\, p(X)p(Y) = 0 .$$

By virtue of equations (3.3), (3.4) and (3.7), we have the following theorem:

Theorem (3.1). If the enveloping manifold M^n be H-structure manifold, its hypersurface M^{n-1} will admit (a, ϵ) metric structure.

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MHD Couette Flow with Pressure Gradient Between Two Parallel Porous Falt Plates

Sutapa Mukherjee

Abstract

A solution in the closed form has been obtained for the flow of a viscous, incompressible and electrically conducting fluid between two parallel porous flat plates in the presence of a transverse magnetic field with suction at the stationary plate and equal injection at the moving plate.

The combined effect of suction and magnetic field on the flow pattern has been studied. It is observed that the transverse magnetic field as well as suction annuls the effect of adverse pressure gradient on the flow.

1. Introduction

Exact solution of the MHD boundary layer equations for the flow of a conducting fluid between two parallel solid plates was given by Hartmann [1]. Dzhorbenadze and Sharikadze [2] studied the flow of a conducting fluid through a channel with porous walls with suction at one wall and equal injection at the other. Mehta and Jain [3] investigated the hydromagnetic flow through a rectangular channel with porous walls in the presence of transverse magnetic field.

Ramamoorthy [4] investigated generalized Couette flow between two porous plates with suction at the stationary plate and injection at the other plate. Rathi [5] discussed the same problem without neglecting the induced magnetic field. Both of them have illustrated the effect of suction and transverse magnetic field on the plane Couette flow without pressure gradient and with pressure decrease.

The chief interest of this paper is to illustrate the effect of suction and transverse magnetic field on the plane Couette flow with pressure rise in which back flow occurs near the stationary plate.

2. Equations of Motion and Solution

Let x and y be the coordinates along and perpendicular to the plates, and u and v the velocity components in the directions of x and y respectively. It is supposed that the distance between the two plates is h, one plate is at rest and the other is moving with velocity U in x-direction. Fluid is being sucked with a constant velocity v_0 at the stationary plate in the presence of a uniform transverse magnetid field H_0 perpendicular to the plates. For infinite flat plates, $\frac{\partial u}{\partial x} = 0$ and the equation

of continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ with condition $y = 0 : v = - v_o$ gives $v = - v_o$ everywhere. To maintain continuous flow an equal amount of fluid is injected into the channel through the moving plate.

The equation for steady two dimensional MHD flow (Ref. 6) reduce to

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho v_o \frac{\partial u}{\partial y} + \frac{1}{\mu e} H_o \frac{\partial B_x}{\partial y} - \frac{\partial p}{\partial x} = 0 \tag{1}$$

and $$\lambda \frac{\partial^2 B_x}{\partial y^2} + v_o \frac{\partial B_x}{\partial y} + H_o \frac{\partial u}{\partial y} = 0 \tag{2}$$

Integrating (2) we get,

$$\lambda \frac{\partial B_x}{\partial y} + B_x v_o + H_o u = \text{Constant} \tag{3}$$

Using Ohm's law we have

$$\lambda \frac{\partial B_x}{\partial y} + B_x v_o + H_o u = \vec{E} \tag{4}$$

In the present problem $\vec{E} = 0$ and hence eqn. (4) reduces to

$$\lambda \frac{\partial B_x}{\partial y} + B_x v_o + H_o u = 0$$

i.e. $$\frac{1}{\mu e} \frac{\partial B_x}{\partial y} = - \sigma (B_x v_o + H_o u) \tag{5}$$

Substituting from (5) into (1),

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho v_o \frac{\partial u}{\partial y} - H_o \sigma (B_x v_o + H_o u) - \frac{\partial p}{\partial x} = 0 \tag{6}$$

Neglecting the induced magnetic field B_x in comparison to the imposed magnetic field H_o, eqn. (6) reduces to

$$\frac{\partial^2 u}{\partial y^2} + \frac{\rho v_o}{\mu} \frac{\partial u}{\partial y} - \frac{H_o^2 \sigma}{\mu} u - \frac{1}{\mu} \frac{\partial p}{\partial x} = 0 \tag{7}$$

Introducing the dimensionless quantities

$$\bar{u} = u/U$$

$$\eta = y/h$$

$$\bar{v}_s = v_o h/\nu$$

$$R_h = H_o h \, (\sigma/\mu)^{1/2}$$

and $$P = \frac{h^2}{\mu U} \frac{\partial p}{\partial x}$$

Equation (7) becomes

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \bar{v}_s \frac{\partial \bar{u}}{\partial \eta} - R_h^{\,2} \, \bar{u} = P \tag{8}$$

The boundary conditions are

$$\eta = 0 \;:\; \bar{u} = 0 \;,$$
$$\eta = 1 \;:\; \bar{u} = 1 \;. \tag{9}$$

The solution of equation (8) satisfying conditions (9) is,

$$\bar{u} = \left[\frac{e^{b_1 \eta} - e^{b_2 \eta}}{e^{b_1} - e^{b_2}}\right] + \frac{P}{R_h^2} \left[\frac{e^{b_1 + b_2 \eta} - e^{b_1 \eta + b_2}}{e^{b_1} - e^{b_2}} + \frac{e^{b_1 \eta} - e^{b_2 \eta}}{e^{b_1} - e^{b_2}} - 1\right] \tag{10}$$

where $b_1 \, (\bar{v}_s, R_h) = \dfrac{-\bar{v}_s + \sqrt{\bar{v}_s^{\,2} + 4R_h^{\,2}}}{2}$

and $b_2 \, (\bar{v}_s, R_h) = \dfrac{-\bar{v}_s - \sqrt{\bar{v}_s^{\,2} + 4R_h^{\,2}}}{2}$

3. Results

Equation (10) gives the velocity distribution for the generalized MHD Couette flow of an electrically conducting fluid between two parallel porous flat plates with pressure gradient and suction at the stationary plate in the presence of a transverse magnetic field.

I. For $R_h \neq 0$ and $\bar{v}_s = 0$,

$$\bar{u} = \left[\frac{e^{R_h\eta} - e^{-R_h\eta}}{e^{R_h} - e^{-R_h}}\right] + \frac{P}{R_h^{\,2}} \cdot \left[\frac{e^{R_h(1-\eta)} - e^{-R_h(1-\eta)}}{e^{R_h} - e^{-R_h}} + \frac{e^{R_h\eta} - e^{-R_h\eta}}{e^{R_h} - e^{-R_h}} - 1\right]$$

which is the solution for generalized MHD Couette flow of a conducting fluid between two parallel solid plates in the presence of a magnetic field perpendicular to the plates.

II. For $R_h = 0$ and $\bar{v}_s = 0$

$$\bar{u} = \eta - \frac{P}{2}(\eta - \eta^2)$$

which is the solution for the generalised plane Couette flow of a non-conducting fluid between two solid plates.

The effect of suction at the stationary plate and the transverse magnetic field on the generalized Couette flow of an electrically conducting fluid has been illustrated by curves in figures 1, 2 and 3 for the three cases of zero pressure gradient, pressure decrease and pressure rise.

It is known that in the case of generalized Couette flow of non-conducting fluid between solid plates with adequate pressure rise in the direction of flow the velocity over a portion of the channel near the stationary plate becomes negative and back flow occurs near the plate.

Suction annuls the effect of adverse pressure gradient and in the absence of magnetic field a value of the suction parameter $\bar{v}_s$ equal to the pressure gradient parameter P gives a linear velocity distribution (Fig. 3) as in the case of flow due to shear only. With $\bar{v}_s$ greater than P an accelerated flow over the whole channel width is obtained even in the presence of a pressure rise.

An externally imposed transverse magnetic field annuls the effect of adverse pressure gradient on the flow and reduces the back flow tending to make it frictionless over a large portion of the channel width. With a strong magnetic field the shear flow due to the moving plate will remain confined within a very thin layer near it. With increasing strength of the transverse magnetic field the velocity gradient at the stationary plate and therefore the skin friction on it decreases.

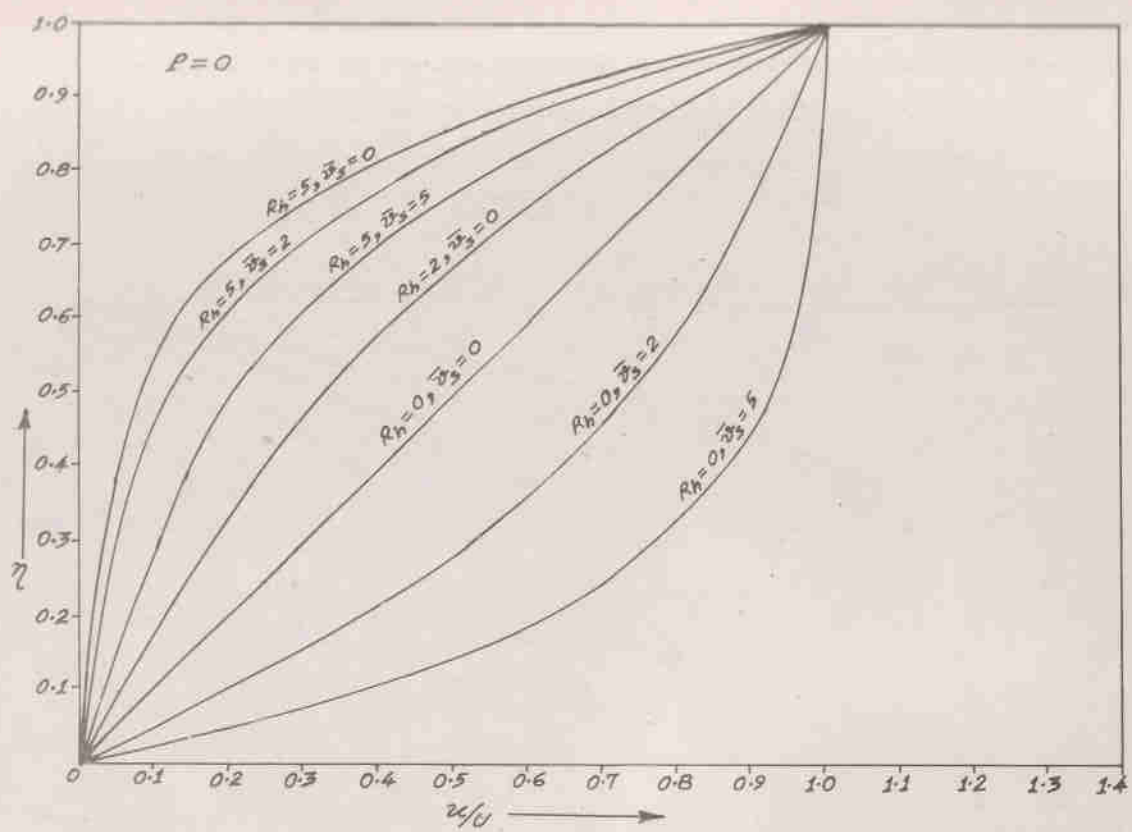

FIGURE – 1

MHD COUETTE FLOW BETWEEN TWO PARALLEL POROUS FLAT PLATES. VELOCITY DISTRIBUTION u/U FOR VARIOUS VALUES OF R_h AND $\bar{v}_s$ IN THE ABSENCE OF PRESSURE GRADIENT ($P=0$).

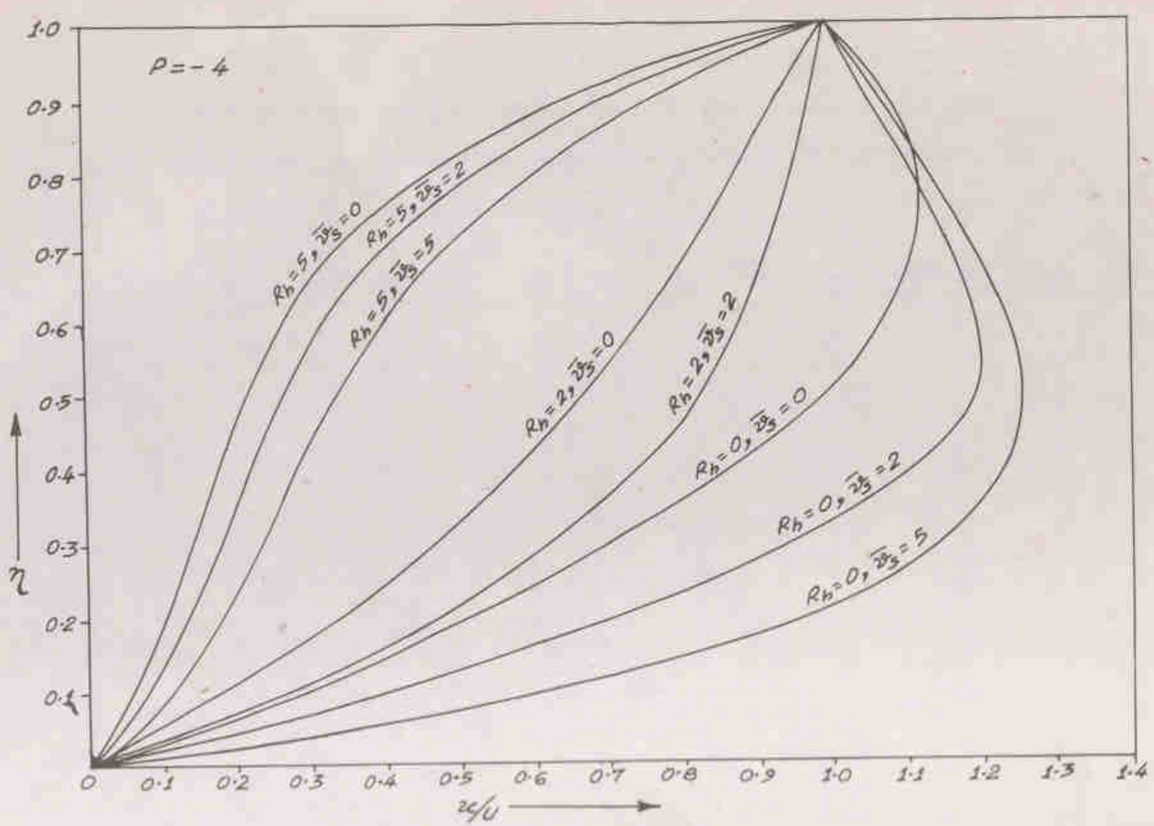

FIGURE - 2

MHD COUETTE FLOW BETWEEN TWO PARALLEL POROUS FLAT PLATES. VELOCITY DISTRIBUTION u/U FOR VARIOUS VALUES OF R_h AND $\bar{v}_s$ IN THE PRESENCE OF A PRESSURE DECREASE $(P=-4)$.

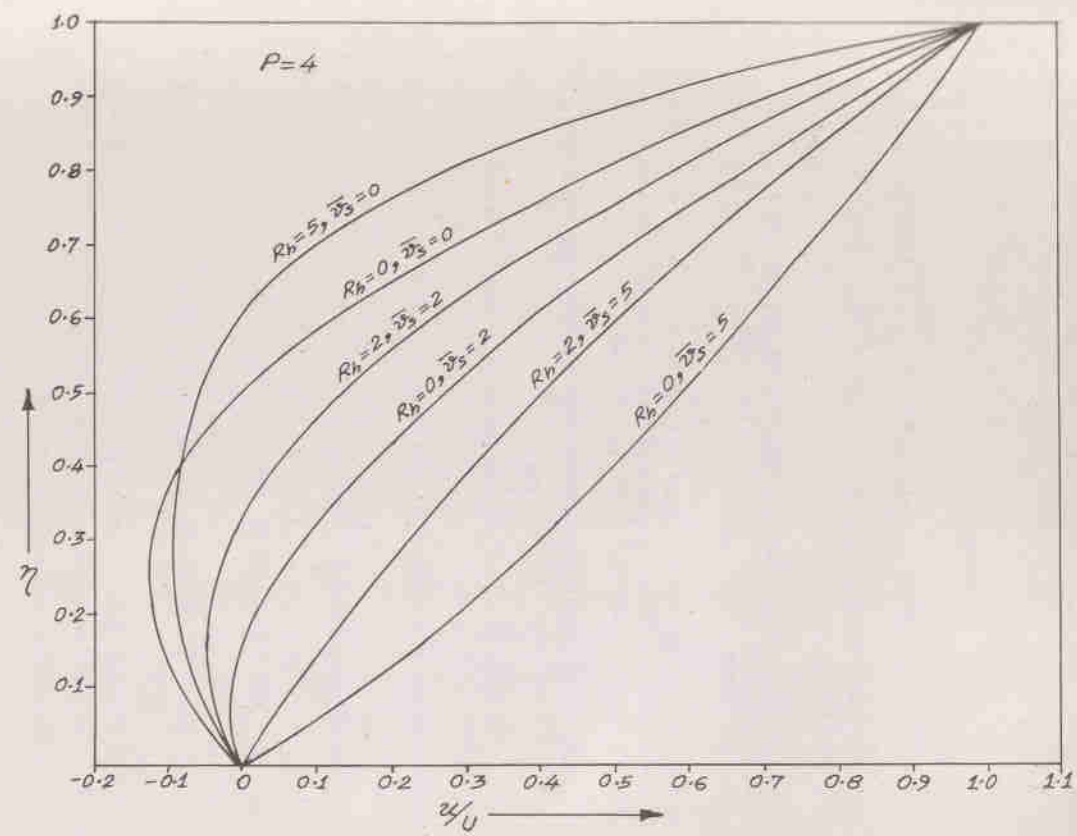

FIGURE - 3

MHD COUETTE FLOW BETWEEN TWO PARALLEL POROUS FLAT PLATES. VELOCITY DISTRIBUTION u/U FOR VARIOUS VALUES OF R_h AND $\bar{v}_s$ IN THE PRESENCE OF A PRESSURE RISE ($P=4$).

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On Generalised Charlier Polynomials

R.M. Shreshtha*

Abstract

Explicit representation, generating function pure recurrence relations and difference equations are obtained for a set of generalised Charlier polynomials.

1. Introduction

The monic Charlier polynomials $C_n^{(a)}(x)$ defined by the explicit formula [1]

$$(1.1) \qquad C_n^{(a)}(x) := \sum_{k=0}^{n} \binom{n}{k} \binom{x}{k} k! \, (-a)^{n-k} \quad (a \neq 0)$$

is known to possess the generating relation [2]

$$(1.2) \qquad e^{-aw} \, (1 + w)^x = \sum_{n=0}^{\infty} C_n^{(a)}(x) \frac{w^n}{n!}$$

The set of Charlier polynomials satisfies the orthogonality relation [1]

$$(1.3) \qquad \int_0^{\infty} C_m^{(a)}(x) \, C_n^{(a)}(x) \, d\,\psi^{(a)}(x) = a^n \, n! \; \delta_{mn} ,$$

where $\psi^{(a)}$ is the step function whose jumps are

$$(1.4) \qquad d\,\psi^{(a)}(x) = \frac{e^{-a} a^x}{x!} \text{ at } x = 0, 1, 2, \ldots .$$

Moreover, the positive definite case occurs for a 0 and in this case, $d\,\psi^{(a)}(x)$ is the Poisson distribution of probability theory. Of various relations satisfied by the set of Charlier polynomials, the following three term recursion formula [1]

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$$(1.5) \qquad C_{n+1}^{(a)}(x) = (x - n - a)\, C_{n}^{(a)}(x) - an\, C_{n-1}^{(a)}(x)$$

is of great importance. Charlier polynomials can also be expressed in terms of Laguerre and Krawtchouk polynomials; and belong to the Hahn class of orthogonal polynomials.

In the present note, we shall obtain analogues of some of the known results for Charlier polynomials.

2. Generalised Charlier Polynomials

The set of generalised Charlier polynomials $C_{n,m}^{(a)}(x)$, for $a \neq 0$ and $m = 1,2,3,\ldots$, may be defined

$$(2.1) \qquad C_{n,m}^{(a)}(x) := (-a)^{n}\, {}_{m+1}F_{0}\left[\Delta(-n;m),\, x;\, -\,;\, m^{m}/a^{m}\right],$$

where $\Delta(-n;m)$ stands for the sequence of m parameters

$$-\frac{n}{m},\ \frac{-n+1}{m},\ \ldots,\ \frac{-n+m-1}{m},$$

and ${}_{m+1}F_{0}$ is a generalised hypergeometric function.

Expanding the hypergeometric series and using the relation [3]

$$(2.2) \qquad (\alpha)_{mk} = m^{mk}\left(\frac{\alpha}{m}\right)_{k}\left(\frac{\alpha+1}{m}\right)_{k}\cdots\left(\frac{\alpha+m-1}{m}\right)_{k},\quad m > 0,\ k \geqslant 0$$

it is easy to obtain the following explicit representation for $C_{n,m}^{(a)}(x)$,

$$(2.3) \qquad C_{n,m}^{(a)}(x) = \sum_{k=0}^{[n/m]} \binom{n}{mk}\binom{x}{k}\, mk!\, (-a)^{n-mk}.$$

A simple generating function for $C_{n,m}^{(a)}(x)$ may be obtained directly from (2.3) in the following form

$$(2.4) \qquad e^{-aw}(1 + w^{m})^{x} = \sum_{n=0}^{\infty} C_{n,m}^{(a)}\, \frac{w^{n}}{n!}, \quad (a \neq 0).$$

Indeed, by virtue of (2.2) and the elementary relations

$$(2.5) \qquad (-n)_{k} = (-1)^{k}\, n!\, /(n-k)!, \quad 0 \leq k \leq n,$$

(2.6) $$(\alpha)_m \, (\alpha + m)_n = (\alpha)_{m+n} \, ,$$

and

(2.7) $$\sum_{n=0}^{\infty} \sum_{k=0}^{[n/m]} A(n-mk,k) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} A(n,k) \, ,$$

we observe that

$$\begin{aligned} e^{-aw} (1 + w^m)^x &= \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \binom{x}{k} \frac{(-a)^n \, w^{n+mk}}{n!} \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{[n/m]} \binom{n}{mk} \binom{x}{k} \, mk! \, \frac{(-a)^{n-mk} \, w^n}{n!} \\ &= \sum_{n=0}^{\infty} C_{n,m}^{(a)}(x) \, \frac{w^n}{n!} \, . \end{aligned}$$

When m = 1, the above results (2.1), (2.3) and (2.4) reduce to well-known results for Charlier polynomials.

3. Pure Rhcurrence Relations and Difference Equations

A pure recurrence relation and a pair of difference equations satisfied by the generalised Charlier polynomials $C_{n,m}^{(a)}(x)$ are as follows:

(3.1) $$\begin{aligned} C_{n+1,m}^{(a)}(x) &= \left\{ mx(n-m+2)_{m-1} - (n-m+1)_m \right\} C_{n-m+1,m}^{(a)}(x) \\ &\quad - a \, C_{n,m}^{(a)}(x) - a(n-m+1)_m \, C_{n-m,m}^{(a)}(x) \, , \end{aligned}$$

(3.2) $$C_{n+1,m}^{(a)}(x) = mx(n-m+2)_{m-1} \, C_{n-m+1,m}^{(a)}(x-1) - a \, C_{n,m}^{(a)}(x) \, ,$$

and

(3.3) $$\begin{aligned} mx(n-m+2)_{m-1} &\left\{ C_{n-m+1,m}^{(a)}(x-1) - C_{n-m+1,m}^{(a)}(x) \right\} \\ &= (n-m+1)_m \left\{ a \, C_{n-m,m}^{(a)}(x) - C_{n-m+1,m}^{(a)}(x) \right\} . \end{aligned}$$

In order to obtain (3.1) and (3.2), we differentiate both sides of (2.4) with respect to w and obtain

$$(3.4) \qquad \left(-a + \frac{mx\, w^{m-1}}{1 + w^{m}}\right) e^{-aw} \, (1 + w^{m})^{x} = \sum_{n=0}^{\infty} C_{n,m}^{(a)}(x) \, \frac{n\, w^{n-1}}{n!}$$

which consequently yield the required results if the coefficients are compared suitably. The last equation is just a simple combination of (3.1) and (3.2).

4. Another Generalisation similar to $C_{n,m}^{(a)}(x)$

A generalisation of the set of Charlier polynomials $C_n^{(a)}(x)$ defined by (1.1) may be proposed in the following form

$$(4.1) \qquad C_n^{(a)}(m:x) := \sum_{k=0}^{[n/m]} \binom{n}{mk} \binom{x}{n - mk} \frac{mk!}{k!} (-a)^{k}$$

It is easy to show that

$$C_n^{(a)}(1:x) = C_n^{(a)}(x)$$

Its generating function is of the form

$$(4.2) \qquad e^{-aw^{m}} \, (1 + w)^{x} = \sum_{n=0}^{\infty} C_n^{(a)}(m:x) \, w^{n} \, ,$$

and possesses properties similar to those of $C_{n,m}^{(a)}(x)$.

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On The Application of Matusita's Measure of Anninity in an Experiment

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Summary

The concept of affinity between two statistical populations was introduced and a number of its properties and applications in statistical decision making were studied by Matusita ([2], [3], [4], [5] and Kirmani ([1]). Here we shall see that the same measure can be used to determine the association between the prior knowledge and the knowledge gained from an experiment.

Let x_1 denotes the knowledge of the state of nature, usually expressed by the knowledge of a finite number of parameters, prior to the experiment and let x_2 be the knowledge after the experiment. There may be a possible association between the knowledge before and after the experiment and one may be interested in measuring how much the knowledge gained after the experiment is related to the knowledge supplied to the experimenter before the experiment is made. In this paper we shall show that the Matusita's measure of affinity can also be used in such problems where the object of experimentation is not to reach decisions but to gain knowledge about the world. The average relationship between the prior and the posterior knowledge for fixed prior knowledge, determines the average amount of association.

Since one of the purposes of performing an experiment is to gain knowledge about the state of nature, knowing something about it prior to the experiment, we suggest in this paper, that one may be interested in performing that experiment for which there is maximum possible association between the knowledge prior and gained after the experiment and will try to continue experimentation until he gets a perfect association between the prior and the gained knowledge.

Let X be a sample space and $x \in X$ be an observation of the experiment. Let $\mathcal{B}$ be a σ-field of subsets of X. Let θ be an element belonging to a space Θ. Then, define a probability measure on $\mathcal{B}$ for every $\theta \in \Theta$. We assume that the probability measures defined on $\mathcal{B}$ are all absolutely continuous with respect to a fixed measure on $\mathcal{B}$ as θ ranges through Θ. Therefore, we can define each probability measure by a probability density function $p(x/\theta)$ such that $\int p(x/\theta)dx$ defines the probability measure of a subset X. Then an experiment E is characterized by $E = \{X, \mathcal{B}, \Theta, P\}$ with $P = \{p(x/\theta)\}$.

Let $p(x)$ and $p(\theta)$ denote the densities of the random variables x and θ respectively. Suppose that Θ is endowed with a σ-field of subsets, then a prior distribution for θ will be a probability measure on this field, which is denoted by $p(\theta)$.

We have

$$p(x) = \int_{\Theta} p(x/\theta)\ p(\theta)d\theta$$

After the experiment has been performed and the value x observed, the posterior distribution of θ is $p(\theta/x)$, where

$$p(\theta/x) = \frac{p(x/\theta)\ p(\theta)}{p(x)}$$

Then, we define the measure of association between the prior knowledge and the posterior knowledge as

$$A(E,x,p(\theta)) = \int_{\Theta} \left\{p(\theta/x).p(\theta)\right\}^{\frac{1}{2}} d\theta \qquad \ldots(1)$$

where the posterior knowledge simply means the knowledge gained after the experiment is performed. If $p(\theta) = 0$, define the integrand to be zero, i.e. when we have no knowledge about the state of nature before conducting the experiment the association is defined to be zero.

The quantity in (1) depends upon x, since some experiments provide greater association with the prior knowledge than the others. Since θ is regarded as a random variable this quantity may be averaged with respect to x according to the probability density $p(x)$.

Definition - The average amount of association between the prior knowledge $p(\theta)$ and the knowledge after the experiment, is,

$$A(E,p(\theta)) = E_x A(E,x,p(\theta)) \qquad \ldots(2)$$

$$= E_x E_\theta \left\{\frac{p(x/\theta)}{p(x)}\right\}^{\frac{1}{2}} \qquad \ldots(3)$$

where E is the expectation operator with respect to the variable denoted in the suffix.

If $p(x,\theta)$ is the joint density for x and θ, we can write it as

$$= \int_X \int_{\Theta} p(x,\theta) \left\{\frac{p(x,\theta)}{p(x)p(\theta)}\right\}^{\frac{1}{2}} dxd\theta \qquad \ldots(4)$$

The expression (4) shows the symmetry between x and θ. Also because the integrand in (4) is positive being the probability densities, we can say that $A(E, p(\theta))$ is positive.

Theorem 1 - $A(E, p(\theta)) = 1$ if $p(x/\theta)$ does not depend on except possibly in a null set for θ.

Proof - Since $p(x/\theta)$ does not depend on the space Θ we have

$$p(x/\theta) = p(x)$$

so that

$$A(E, p(\theta)) = E_x E_\theta \left\{ \frac{p(x)}{p(x)} \right\}^{\frac{1}{2}}$$

$$= 1 \text{ since } E(1) = 1$$

Hence, the result is proved.

This is the case when $p(x/\theta) = p(x)$, or $p(\theta/x) = p(\theta)$, that is posterior distribution of θ is exactly equal to the prior distribution. In this case we can say that there is perfect association between the prior knowledge and the posterior knowledge about the state of nature.

New suppose that the observations x in an experiment consist of a pair of observations x_1, x_2. That is every $x \in X$ is an ordered pair (x_1, x_2) with $x_i \in X_i$, $i = 1, 2$. Let β_i be the σ-field over X_i induced by β by the transformation $x_i = x_i(x)$ and let P_i be the set of probability densities $p(x_i/\theta)$ of the observations x_i, $i = 1, 2$. We can say that the experiment E is the sum of the two experiments E_1 and E_2 with $E_i = \{X_i, \beta_i, \Theta, P_i\}$ $i = 1, 2$. The experiment $E_2(x_1) = \{X_2, \beta_2, \Theta, P_2(x_1)\}$ is also considered with $P_2(x_1)$ as the set of probability densities $p(x_2/\theta, x_1)$.

We note that, since $p(\theta/x_1)$ is the posterior distribution of θ after x_1 has been observed, $A(E_2(x_1), p(\theta/x_1))$ defines the average amount of association between the prior knowledge $p(\theta/x_1)$ and the knowledge gained from the experiment E_2 by observing x_2 after E_1 has been performed and x_1 observed. The average of it over x_1 gives the average association between the prior knowledge and the knowledge provided by the experiment E_2 after E_1 has been performed. We will denote it as

$$A(E_2/E_1, p(\theta/x_1)) = E_{x_1} A(E_2(x_1), p(\theta/x_1))$$

$$= E_{x_1} E_{x_2} E_\theta \left\{ \frac{p(x_2/\theta, x_1)}{p(x_2/x_1)} \right\}^{\frac{1}{2}} \qquad \ldots(5)$$

Theorem 2 - $A(E_1, p(\theta)), A(E_2/E_1, p(\theta/x_1)) = A(E, p(\theta))$

Proof - We have

$$A(E_1, p(\theta)) = E_{x_1} E_\theta \left\{ \frac{p(x_1/\theta)}{p(x_1)} \right\}^{\frac{1}{2}}$$

$$= E_{x_1} E_{x_2} E_\theta \left\{ \frac{p(x_1/\theta)}{p(x_1)} \right\}^{\frac{1}{2}} \qquad \ldots(6)$$

and

$$A(E_2/E_1, p(\theta/x_1) = E_{x_1} E_{x_2} E_\theta \left\{ \frac{p(x_2/\theta, x_1)}{p(x_2/x_1)} \right\}^{\frac{1}{2}} \qquad \ldots(7)$$

From (6) and (7), we get

$$A(E_1, p(\theta)).\ A(E_2/E_1, p(\theta/x_1))$$

$$= E_{x_1} E_{x_2} E_\theta \left\{ \frac{p(x_1/\theta)}{p(x_1)} \right\}^{\frac{1}{2}} \cdot \left\{ \frac{p(x_2/\theta, x_1)}{p(x_2/x_1)} \right\}^{\frac{1}{2}}$$

$$= E_{x_1} E_{x_2} E_\theta \left\{ \frac{p(x_1, \theta)}{p(\theta)\ p(x_1)} \cdot \frac{p(x_2, \theta, x_1)\ p(x_1)}{p(\theta, x_1)\ p(x_2, x_1)} \right\}^{\frac{1}{2}}$$

$$= E_{x_1} E_{x_2} E_\theta \left\{ \frac{p(x_1, x_2/\theta)}{p(x_1, x_2)} \right\}^{\frac{1}{2}}$$

$$= A(E, p(\theta))$$

Hence the theorem is proved.

Corollary - If x_1 is sufficient for x in the Neyman Fisher sense, then

$$A(E_1, p(\theta)) = A(E, p(\theta)).$$

Proof - If x_1 is sufficient for x, by the factorization theorem we can easily see that $p(x_2/\theta, x_1)$ does not depend on θ and so according to Theorem 1, it follows that $A(E_2E_1, p(\theta/x_1)) = 1$. Hence from the statement of the Theorem 2 the corollary is established.

The corollary says that the association between the prior knowledge and the knowledge gained from the experiment is not changed after performing E_2 if the observation x_1 in an experiment is a sufficient statistics.

Theorem 2 can be generalized to a finite number of experiments with common Θ.

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On A Structure $F_{\alpha, \lambda}$ Satisfying $f^2 + \alpha f - \lambda^2 I = 0$

V.C. Gupta and Jaya Pant

Summary

Sinha and Sharma [3] have defined special quadratic structure f satisfying $f^2 + \alpha f - I = 0$. In this paper, we have defined and studied the structure $f_{\alpha,\lambda}$ satisfying $f^2 + \alpha f - \lambda^2 I = 0$, where f is a non-null tensor field of the type (1, 1) and λ is a complex number, not equal to zero. This is a generalization of the π-structure [1]. First we have defined the projection operators and then the existence and integrability conditions of this structure have been obtained.

1. Preliminaries

Let us consider an n-dimensional real differentiable manifold M of class C^∞ endowed with a non-null tensor field f of type (1, 1) satisfying the algebraic equation

$$(1.1) \qquad f^2 + \alpha f - \lambda^2 I = 0,$$

where λ is a complex number not equal to zero and $\alpha \in R$ (the set of real numbers). If X denotes an arbitrary vector field in the $C^\infty(M)$ module $\mathfrak{X}(M)$ of M, where $\mathfrak{X}(M)$ is the algebra over the vector fields in M; then (1.1) can also be expressed as

$$(1.2) \qquad \bar{\bar{X}} + \alpha \bar{X} - \lambda^2 X = 0,$$

where $\bar{X} \overset{\text{def}}{=} f(X)$. The rank of f can be easily shown to be equal to n [3]. We call the structure satisfying (1.1) a "$f_{\alpha,\lambda}$-structure." Let 'e' be an eigen value of f, then e is a root of the quadratic equation

$$(1.3) \qquad e^2 + \alpha e - \lambda^2 I = 0.$$

The roots of (1.3) are

$$+\tfrac{1}{2}\left(-\alpha \pm \sqrt{\alpha^2 + 4\lambda^2}\right),$$

which may be taken to be e^{θ} and $e^{-\theta}$ if we set $\lambda \sin h\theta$ for $-\frac{1}{2}\alpha$. Let us suppose that the multiplicities of e^{θ} and $e^{-\theta}$ are p and q respectively, such that $p + q = n$.

2. Distributions of M with $f_{\alpha,\lambda}$ -Structure

Let us define a pair of projection operators s and t on the tangent space at each point of M as follows:

$$s(X) = \frac{1}{1+e^{2\theta}} \left(X + \frac{e^{\theta}}{\lambda}\bar{X}\right), \tag{2.1}$$

$$t(X) = \frac{1}{1+e^{-2\theta}} \left(X - \frac{e^{-\theta}}{\lambda}\bar{X}\right). \tag{2.2}$$

Theorem (2.1). For a structure $f_{\alpha,\lambda}$ satisfying (1.1), the projection operators s and t defined by (2.1) and (2.2) and applied to the tangent space at each point of M are complementary projection operators.

Proof. In consequence of (1.2), (2.1) and (2.2), we have

$$\begin{aligned}
s^2(X) &= s(s(X)), \\
&= \frac{1}{\lambda(1+e^{2\theta})}\left\{\lambda s(X) + \frac{e^{\theta}}{\lambda(1+e^{2\theta})}\left(\lambda\bar{X} + e^{\theta}\bar{\bar{X}}\right)\right\}, \\
&= \frac{1}{\lambda(1+e^{2\theta})}\left\{\lambda s(X) + \frac{e^{\theta}}{\lambda(1+e^{2\theta})}\left(\lambda\bar{X}e^{-\theta} + \lambda^2 X - \alpha\bar{X}\right)\right\}, \\
&= \frac{1}{\lambda(1+e^{2\theta})}\left\{\lambda s(X) + \frac{e^{2\theta}}{(1+e^{2\theta})}\left(\bar{X}e^{\theta} + \lambda X\right)\right\}, \\
&= \frac{1}{\lambda(1+e^{2\theta})}\left\{\lambda s(X) + e^{2\theta}\lambda s(X)\right\}, \\
&= s(X)
\end{aligned}$$

Similarly, we can show that $t^2(X) = t(X)$.

Also we have

$$t(s(X)) = t\left\{\frac{1}{\lambda(1+e^{2\theta})}\quad(\lambda X + e^{\theta}\,\bar{X})\right\},$$

$$= \frac{1}{\lambda(1+e^{2\theta})}\left\{\lambda t(X) + e^{\theta}\,t(\bar{X})\right\},$$

$$= 0\ .$$

Next we have

$$s(X) + t(X) = \frac{1}{\lambda(1+e^{2\theta})}\ (\lambda X + e^{\theta}\,\bar{X})\ -$$

$$= \frac{1}{\lambda(1+e^{-2\theta})}\ (\lambda X - e^{-\theta}\,\bar{X})$$

$$= X\ .$$

Thus s and t satisfy the following relations:

$$\text{(2.3)}\qquad s + t = I\ ,\quad st = 0\ ,\quad s^2 = s\ ,\quad t^2 = t\ .$$

This proves the theorem.

Let S and T be the complementary distributions of dimensions p and q corresponding to the projection operators s and t respectively. Let $\left\{{P \atop a}\right\}$, $a = 1, 2, \ldots, p$ and $\left\{{Q \atop x}\right\}$, $x = 1, 2, \ldots, q$ be linearly independent set of vectors spanning S and T respectively. It can be easily shown that the set $\left\{{P \atop a}, {x \atop q}\right\}$ be the inverse set of the set $\left\{{P \atop a}, {a \atop x}\right\}$. Then we can prove that

$$\text{(2.4)}\qquad \overset{a}{p}(\underset{b}{P}) = \delta^{a}_{b}\qquad \overset{a}{p}(\underset{x}{Q}) = 0$$

$$\text{(2.5)}\qquad \overset{a}{q}(\underset{a}{P}) = 0\ ,\qquad \overset{x}{q}(\underset{y}{Q}) = \delta^{x}_{y}$$

$$\text{(2.6)}\qquad \overset{a}{p}(X)\underset{a}{P} + \overset{x}{q}(X)\underset{x}{Q} = X\ .$$

Theorem (2.2). We have

$$(2.7) \qquad s(X) = \overset{x}{p}(X)P_{a}\ , \quad t(X) = \overset{x}{q}(X)Q_{x}\ .$$

Proof. By virtue of (2.4), (2.5) and (2.6), we have

$$s(X) = s\left\{\overset{a}{p}(X)P_{a} + \overset{x}{q}(X)Q_{x}\right\}$$

$$= \overset{a}{p}(X)P_{a}\ .$$

Similarly, the remaining part of the theorem can be proved.

Theorem (2.3). In order that M admits the structure $f_{\alpha,\lambda}$ satisfying $f^2 + \alpha f - \lambda^2 I = 0$, it is necessary and sufficient that the distributions S and T have no direction in common and span together a linear space of dimension n.

Proof.

Let M be equipped with a structure $f_{\alpha,\lambda}$ satisfying $f^2 + \alpha f - \lambda^2 I = 0$. Then in view of projection operators s and t defined by (2.1) and (2.2), we get a pair of global distributions S and T having no direction in common. Moreover, if P and Q are linearly independent set of vectors spanning S and T, then it can be easily shown that the set $\left\{P_a, Q_x\right\}$ is a linearly independent set. Hence the distributions S and T span a linear space of dimension n.

Conversely, let there exist a pair of distributions S and T having no direction in common and spanning the linear space of dimension n. Let $\left\{P_a\right\}$ be the set of p linearly independent vectors in S and $\left\{Q_x\right\}$ be the set of q linearly independent vectors in T. Then $\left\{P_a, Q_x\right\}$ is a linearly independent set. Let us define the inverse set $\left\{\overset{a}{p}, \overset{x}{q}\right\}$ as in (2.4), (2.5) and (2.6). We set

$$\bar{X} = \lambda e^{\theta}\, \overset{a}{p}(X)P_{a} - \lambda e^{-\theta}\, \overset{x}{q}(X)Q_{x}\ .$$

Therefore in consequence of (2.4), (2.5) and (2.6), we have

$$\bar{\bar{X}} = \lambda e^{\theta} \overset{a}{p}\left\{\lambda e^{\theta} \overset{b}{p}(X)P_b - \lambda e^{-\theta} \overset{y}{q}(X)Q_y\right\} P_a -$$

$$- \lambda e^{-\theta} \overset{x}{q}\left\{\lambda e^{\theta} \overset{b}{p}(X)P_b - \lambda e^{-\theta} \overset{y}{q}(X)Q_y\right\} Q_x$$

$$= \lambda^2 e^{2\theta} \overset{a}{p}(X)P_a + \lambda^2 e^{-2\theta} \overset{x}{q}(X)Q_x ,$$

$$= \lambda^2 \left\{e^{2\theta} \overset{a}{p}(X)P_a + e^{-2\theta} \overset{x}{q}(X)Q_x\right\}.$$

Hence

$$\bar{\bar{X}} + \alpha \bar{X} = \overset{a}{p}(X)P_a\left\{\lambda^2 e^{2\theta} + \alpha\lambda e^{\theta}\right\} + \overset{x}{q}(X)Q_x\left\{\lambda^2 e^{-2\theta} - \alpha\lambda e^{-\theta}\right\},$$

$$= \lambda^2 \left\{\overset{a}{p}(X)P_a + \overset{x}{q}(X)Q_x\right\}$$

$$= \lambda^2 X.$$

Thus M admits the $f_{\alpha,\lambda}$-structure satisfying $f^2 + \alpha f - \lambda^2 I = 0$.

3. Nijenhuis Tensor of $f_{\alpha,\lambda}$-Structure

The Nijenhuis tensor of the structure $f_{\alpha,\lambda}$ satisfying (1.1) is the skew-symmetric tensor of type (1,2) given by [4]

$$N(X,Y) = [\bar{X},\bar{Y}] + \overline{\overline{[X,Y]}} - \overline{[\bar{X},Y]} - \overline{[X,\bar{Y}]} , \tag{3.1}$$

for arbitrary vector fields X,Y in M.

Theorem (3.1). We have

$$N(X,\bar{Y}) = N(\bar{X},Y) , \tag{3.2}$$

$$(3.3) \qquad N(\bar{X},\bar{Y}) = \lambda^2 N(X,Y) - \alpha N(X,\bar{Y}) ,$$

$$(3.4) \qquad N(\bar{X},Y) = - \alpha N(X,Y) - \overline{N(X,Y)} .$$

<u>Proof</u>. Barring X in (3.1), we have

$$N(\bar{X},Y) = [\bar{\bar{X}},\bar{Y}] + [\overline{\overline{\bar{X},Y}}] - [\overline{\bar{\bar{X}},Y}] - [\overline{\bar{X},\bar{Y}}],$$

which in view of (1.2) reduces to

$$(3.5) \qquad N(\bar{X},Y) = \lambda^2 [X,\bar{Y}] - \alpha [\bar{X},\bar{Y}] + \lambda^2 [\bar{X},Y]$$
$$- \lambda^2 [\overline{X,Y}] - [\overline{\bar{X},\bar{Y}}].$$

Barring Y in (3.1) and using (1.2), we have

$$(3.6) \qquad N(X,\bar{Y}) = \lambda^2 [X,\bar{Y}] + \lambda^2 [\bar{X},Y] - \alpha [\bar{X},\bar{Y}]$$
$$- [\bar{X},\bar{Y}] - \lambda^2 [X,Y] .$$

From (3.5) and (3.6) we obtain (3.2).

Barring X and Y in (3.1) and using (1.2), we have

$$(3.7) \qquad N(\bar{X},\bar{Y}) = \lambda^4 [X,Y] - \lambda^2 \alpha [X,\bar{Y}] - \lambda^2 \alpha [\bar{X},Y]$$
$$+ \alpha^2 [\bar{X},\bar{Y}] + \lambda^2 [\bar{X},\bar{Y}] - \lambda^2 [\overline{X,\bar{Y}}]$$
$$+ \alpha [\overline{\bar{X},\bar{Y}}] - \lambda^2 [\overline{\bar{X},Y}].$$

In consequence of (3.5), we have

$$(3.8) \qquad \alpha N(X,\bar{Y}) = \alpha \lambda^2 [\bar{X},Y] - \alpha^2 [\bar{X},\bar{Y}] + \alpha \lambda^2 [X,\bar{Y}] -$$
$$- \alpha [\overline{\bar{X},\bar{Y}}] - \alpha \lambda^2 [\overline{X,Y}].$$

From (3.1), (3.7) and (3.8), we get (3.3).

Equation (3.4) follows from (3.1) and (3.5).

4. Integrability Conditions

Theorem (4.1). The distribution S is integrable, if and only if

(4.1) $\quad (d\ t)\ (X,Y) = 0\ .$

Proof. Let us assume that S is integrable, then

$$X,Y \in S \Longrightarrow [X,Y] \in S.$$

Therefore

$$s(X) = X,\ s(Y) = Y,\quad t(X) = 0,\quad t(Y) = 0,\ t([X,Y]) = 0.$$

We have [2]

$$(d\ t)\ (X,Y) = Xt(Y) - Yt(X) - t([X,Y]),$$

which by virtue of (4.2) yields

$$(d\ t)\ (X,Y) = 0.$$

Conversely, suppose that $(d\ t)(X,Y) = 0$, where $X,Y \in S$. Then $t([X,Y]) = 0$, since $t(X) = 0$ and $t(Y) = 0$.

Further, from (2.1), we have

$$\begin{aligned} s(X,Y) &= \frac{1}{1+e^{2\theta}}\left\{ [X,Y] + \frac{e^{\theta}}{\lambda}\ (\overline{[X,Y]}) \right\}, \\ &= \frac{1}{\lambda(1+e^{2\theta})}\left\{ \lambda[X,Y] + e^{\theta}\ \overline{[X,Y]} \right\}. \end{aligned} \tag{4.3}$$

Also, we have

$$\begin{aligned} t([X,Y]) &= \frac{1}{1+e^{-2\theta}}\ \left([X,Y] - \frac{e^{-\theta}}{\lambda}\ \overline{[X,Y]}\right), \\ &= \frac{1}{\lambda(1+e^{-2\theta})}\left\{\lambda[X,Y] - e^{-\theta}\ \overline{[X,Y]}\right\} \end{aligned} \tag{4.4}$$

Thus from (4.3) and (4.4), we have

$$s([X,Y]) = [X,Y].$$

Hence $[X,Y] \in S$.

Thus we have shown that $X, Y \in S \Rightarrow [X,Y] \in S$

It follows that S is integrable.

Theorem (4.2). The distribution T is integrable, if and only if

$$(d\ s)(X,Y) = 0 . \tag{4.5}$$

Proof. The proof follows from the pattern of the proof of theorem (4.1).

Theorem (4.3). A necessary and sufficient condition for the structure $f_{\alpha,\lambda}$ satisfying (1.1) to be integrable is that the Nijenhuis tensor of $f_{\alpha,\lambda}$ vanishes identically.

Proof. By theorem (4.1), the distribution S is integrable if and only if $(d\ t)(X,Y) = 0$, where X,Y satisfy $s(X) = X$. Therefore, the above condition is equivalent to $(d\ t)(s(X), s(Y)) = 0$.

Since $ts = 0$, therefore $t\,[s(X), s(Y)] = 0$. In consequence of (2.1) and (2.2), the above equation takes the form

$$\left[\lambda X + e^{\theta}\bar{X},\ \lambda Y + e^{\theta}\bar{Y}\right] - \frac{e^{-\theta}}{\lambda}\overline{\left[\lambda X + e^{\theta}\bar{X},\ \lambda Y + e^{\theta}\bar{Y}\right]} = 0$$

or equivalently,

$$\left[\overline{\bar{X},\bar{Y}}\right] + e^{\theta}\left(\lambda\left[X,\bar{Y}\right] + \lambda\left[\bar{X},Y\right] - \lambda\left[\overline{X,Y}\right] - \frac{1}{\lambda}\left[\overline{\bar{X},\bar{Y}}\right]\right)$$

$$+ e^{\theta}\left(\left[\bar{X},\bar{Y}\right]\right) - \left[\overline{X,\bar{Y}}\right] - \left[\overline{\bar{X},Y}\right] = 0 ,$$

which by virtue of (3.1) and (3.5) reduces to

$$N(X,Y) - \frac{e^{-\theta}}{\lambda}\overline{N(X,Y)} = 0 . \tag{4.6}$$

Proceeding in a similar manner and using theorem (4.2), we can show that the necessary and sufficient condition for the distribution T to be completely integrable is that

$$N(X,Y) + \frac{e^{\theta}}{\lambda}\overline{N(X,Y)} = 0 \tag{4.7}$$

The conditions (4.6) and (4.7) imply that for the complete integrability of both the distributions S and T, $N = 0$. Thus the structure $f_{\alpha,\lambda}$ is integrable if and only if $N = 0$.

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Stochastic Production -- Inventory Model (SPIM) For Deteriorating Items

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Abstract

Sethi and Thompson [3] have developed a SPIM for non-deteriorating items and derived closed form expressions for optimal production rates when the demand rate is constant by minimising a quadratic loss function. A similar model for an inventory of items that deteriorate after some time from their arrival in the inventory is presented in this paper. Closed form expressions for the optimal production rates for finite and infinite horizon versions of SPIM are derived for the general and specific deterioration functions when the demand rate is assumed constant. For deterministic demand, the corresponding general optimal production rate is obtained. The case of discounted objective function is also considered.

1. Introduction

We consider a general production inventory model for items that deteriorate in the inventory after some time from their arrival in the inventory. The deterioration takes place according to an instantaneous deterioration rate $\phi(t)$, $0 \leq \phi(t) < 1$. We derive closed form expressions for the optimal production rates for both finite and infinite horizon versions of the SPIM when the deterioration rate is specified. We consider the cases separately when the demand rate is constant and when it is deterministic.

The model considered here is developed under the following general assumptions: (i) The production process involves the production of a homogeneous good and have an inventory system. (ii) The demand rate of S units per unit of time is known. (iii) Deterioration of items starts after some time $\gamma\ (> 0)$ of arrival in the inventory. (iv) At time t, a fraction $\phi(t)$ of the on-hand inventory deteriorates. $\phi(t)$ is assumed integrable.

2. The Model

Let $x'(t')$, $u'(t')$ and x_1, u_1 be the inventory level and production level at time t' and Y respectively. Let S' be the demand rate at time t'. The change in the inventory level is then described by Ito's stochastic differential equation

(1) $$dx' = (u' - S')dt' + \sigma\, dz$$

with initial condition $x'(Y) = x_1$, where σ is the constant diffusion coefficient and $dZ(t')$ may be expressed as $\xi(dt')$ where $\xi(dt')$ is a SP with $E\left[\xi(dt')\right] = 0$, $E\left[\xi^2(dt')\right] = \sigma^2\,dt'$ and $E\left[\xi^n(dt')\right] = o(dt')$ for all $n > 2$. We wish to control $u'(t')$, $Y \leq t' \leq T'$ so as to guide the production process in such a way as to adjust the inventory level initially at x_1 towards S'.

Let h, c, B_1 and B respectively denote the holding cost, production cost, salvage value per deteriorating item and salvage value per unit terminal inventory respectively. Then for $t = t' - Y$ and $T = T' - Y$, the value function to be maximised is

$$(2) \quad V(t,x) = \max_u \; E\left[-\int_t^T (cu^2 + h\,\psi^2 x^2)dt + \int_t^T B_1\,\emptyset\,x\,dt + Bx(T)\right]$$

with the terminal condition

$$(3) \quad V(T,x) = Bx(T).$$

If $V(t,x)$ is continuously differentiable in t and twice continuously differentiable in x then using Ito's differentiation rule and writing $x(t) = x$, we obtain the following Hamilton-Jacobi-Bellman equation

$$(4) \quad 0 = \max_u \left\{-(cu^2 + h\,\psi^2 x^2) + B_1\emptyset\,x + V_t - SV_x + \tfrac{1}{2}\sigma^2 V_{xx}\right\}$$

where $S = S' - u_1$. Differentiating (4) w.r.t. u and equating to zero yields the optimal production rate as

$$(5) \quad u = \frac{V_x}{2c}$$

which on substitution into (4) and using the terminal condition (3) yields the PDE for the optimal production rate as

$$(6) \quad 0 = \frac{Vx^2}{4c} - (h\,\psi^2 x^2 - B_1\emptyset\,x) + V_t - SV_x + \tfrac{1}{2}\sigma^2 V_{xx}.$$

3. SPIM When Demand Rate is Constant

Assume that $V(t,x)$ has a separable form

$$(7) \quad V(t,x) = Q(t)\,(\psi x)^2 + R(t)\,(\psi x) + M(t).$$ Then we can find V_t, V_x and V_{xx} which on substitution into (6) yields for any x,

$$\text{(8)} \quad \frac{(\psi^2 Q)^2}{c} - h\psi^2 + \psi^2 \dot{Q} + 2\psi Q \dot{\psi} = 0 \text{ with b.c. } Q(T) = 0$$

$$\text{(9)} \quad \frac{\psi^2 Q}{c} \quad \psi R + \psi \dot{R} + \psi R + B_1 \emptyset - 2S\, \psi^2 Q = 0 \text{ with b.c. } R(T)\psi(T) = B$$

$$\text{(10)} \quad \frac{R^2 \psi^2}{4c} + \dot{M} - SR\psi + \sigma^2 \psi^2 Q = 0 \text{ with b.c. } M(T) = 0$$

where the boundary conditions are obtained by comparing (7) at $t = T$ with (3).

The solution to (8) yields

$$\text{(11)} \qquad Q = \frac{\sqrt{ch}}{\psi} \cdot \frac{y-1}{y+1} \qquad \text{where}$$

$$\text{(12)} \qquad y = \exp\left\{-2\sqrt{\frac{h}{c}} \int_t^T \psi \, du\right\}$$

The solution to (9) yields

$$\text{(13)} \quad R = \frac{c}{\psi}\left[2S + \left(\frac{B}{c} - 2S\right)\exp\left\{\frac{1}{c}\int_t^T \psi^2 \, Q ds\right\}\right] + \frac{B_1}{\psi}\int_t^T \emptyset(s)\exp\left\{\frac{1}{c}\int_t^s \psi^2 Q du\right\} ds.$$

Using (12) and (13) on (5) we get the optimal production rate (OPR) as

$$\text{(14)} \quad u^* = S + \sqrt{\frac{h}{c}} \cdot \frac{y-1}{y+1}(\psi x) + \frac{1}{2}\left(\frac{B}{c} - 2S\right)\exp\left\{\frac{1}{c}\int_t^T \psi^2 Q ds\right\} + \frac{B_1}{2c}\int_t^T \emptyset(s)\exp\left\{\frac{1}{c}\int_t^s \psi^2 Q du\right\} ds$$

$$\text{(15)} \quad = S + \sqrt{\frac{h}{c}} \cdot \frac{y-1}{y+1}\psi x + \left(\frac{B}{c} - 2S\right)\frac{\sqrt{y}}{y+1} + \frac{B_1}{2c}\frac{\sqrt{y}}{y+1}\int_t^T \emptyset(s)\,\frac{y_s+1}{\sqrt{y_s}}\, ds\ .$$

Since $\emptyset(t)$ is the deterioration rate, it is related to the deterioration p.d.f. $f(t)$ as $\emptyset(t) = f(t)/(1-F(t))$. If we denote by

$$K(\theta) = \exp\left\{\theta\int_0^T \psi\, du\right\} \text{ and } M(\theta) = \frac{1}{\theta}\int_t^T e^{\theta s} f(s)ds$$

with $f(t) = \phi(t).\ \exp\left\{-\int_0^t \phi\ (s)ds\right\}$ then,

$$\int_t^T \phi(s)\ \frac{y_s + 1}{\sqrt{y}s}\ ds = K^{-1}(\theta)M(\theta) + K(\theta)M(-\theta)$$

where $O = \sqrt{(h/c)}$, and (15) becomes

$$(16)\quad u^* = S + \sqrt{(h/c)}\ \frac{y-1}{y+1}\ \psi x + (\frac{B}{c} - 2S)\frac{\sqrt{y}}{y+1}$$
$$+ \frac{B_1}{2c} \cdot \frac{\sqrt{y}}{y+1}\left[K^{-1}(\theta)M(\theta) + K(\theta)M(-\theta)\right]$$

Thus the OPR depends on the demand rate, the on-hand inventory level, the decay rate and the distance from the planning horizon.

For no deterioration of items, (16) reduces to

$$(17)\quad u^* = S + \sqrt{(h/c)}\ \frac{y-1}{y+1}\ x + (\frac{B}{c} - 2S)\frac{\sqrt{y}}{y+1}$$

so that the change in the OPR due to the presence of deterioration rate is

$$(18)\quad \Delta u^* = -\ \phi.\sqrt{(h/c)}\ \frac{y-1}{y+1}\ x + \frac{B_1}{2c}\ \frac{\sqrt{y}}{y+1}\left[K^{-1}\ (\theta)M(\theta) + K(\theta)M(-\theta)\right]$$

since $y-1 < 0$, $\Delta u^* \geq 0$, the equality holding at $t = T$. Thus greater the planning horizon and greater the deterioration rate, greater should be the OPR as one would intuitively expect.

<u>Special Cases:</u>

i) For h=c=1, (17) reduces to (30) of [3].

ii) For $\phi(t) = \phi$, a constant, the OPR is given by

$$(19)\quad u^* = S + (1-\phi)\ \sqrt{(h/c)}\ \frac{y-1}{y+1}\ x\ (\frac{B}{c} - 2S)\frac{\sqrt{y}}{y+1}$$
$$+ \frac{\phi}{1-\phi}\ \frac{B_1}{2\sqrt{hc}} \cdot \frac{y-1}{y+1} \text{ where } y = e^{-2\sqrt{(h/c)}(1-\phi)(T-t)}$$

(iii) For $f(t) = \beta t^{\beta-1}$, $\alpha > 0$, $\beta > 0$ and $0 < t < T$

$$\text{i.e. } \phi(t) = \frac{\alpha\beta\, t^{\beta-1}}{1-\alpha t^{\beta}},$$

$$(20)\quad u^* = S + \sqrt{(h/c)}\,\frac{y-1}{y+1}\left(1-\frac{\beta t^{\beta-1}}{1-\alpha t^{\beta}}\right)x + \left(\frac{B}{c} - 2S\right)\frac{\sqrt{y}}{y+1}$$

$$+ \frac{B_1}{2c}\frac{\sqrt{y}}{y+1}\,\frac{e^{-\theta T}}{(1-\alpha T^{\beta})^{\theta}}\cdot\frac{\alpha}{\theta}\left\{T^{\beta}\,{}_1F_1(\beta,\beta+1,\theta T) - t^{\beta}\,{}_1F_1(\beta,\beta+1,\theta t)\right\}$$

$$- e^{\theta T}(1-\alpha T^{\beta})^{\theta}\,\frac{\alpha}{\theta}\left\{T^{\beta}\,{}_1F_1(\beta,\beta+1,-\theta T) - t^{\beta}\,{}_1F_1(\beta,\beta+1,-\theta t)\right\}$$

where, $y = \exp\left\{-2\theta(T-t)\right\}\left[(1-\alpha t^{\beta})/(1-\alpha T^{\beta})\right]^{2\theta}$ and

${}_1F_1(p,p+1,x) = \sum_{r=0}^{\infty} \frac{\Gamma(r+p)/\Gamma(p)}{\Gamma(r+p+1)/\Gamma(p+1)}\cdot\frac{x^r}{r!}$ is a hypergeometric function of Gauss form and are tabulated in [2].

(iv) If the deterioration p.d.f. is given by $f(t) = \alpha\beta e^{\beta t}$

$$\text{i.e. } \phi(t) = \frac{\alpha\beta\, e^{\beta t}}{1-\alpha(e^{\beta t}-1)}$$

then $y = \exp\left\{-2\theta(T-t)\right\}\left[\left(1-\alpha(e^{\beta t}-1)\right) / \left(1-\alpha(e^{\beta T}-1)\right)\right]$

and $u^* = S + \left(1 - \frac{\alpha\beta e^{\beta t}}{1-\alpha(e^{\beta t}-1)}\right)\sqrt{(h/c)}\,\frac{y-1}{y+1}\,x + \left(\frac{B}{c} - 2S\right)\frac{\sqrt{y}}{y+1}$

$$(21)\qquad + \frac{B_1}{2c}\frac{\sqrt{y}}{y+1}\left[\frac{e^{-\theta T}}{1-\alpha(e^{\beta T}-1)}\,\frac{\alpha\beta}{\theta(\theta+\beta)}\left\{e^{(\theta+\beta)T} - e^{(\theta+\beta)t}\right\}\right.$$

$$\left. - \frac{e^{\theta T}\left(1-\alpha(e^{\beta T}-1)\right)\alpha\beta}{\theta(\theta+\beta)}\left\{e^{(\beta-\theta)T} - e^{(\beta-\theta)t}\right\}\right]$$

Asymptotic Behaviour of the OPR

If we examine the asymptotic behaviour of the OPR for a long process ($T \rightarrow \infty$), we find that starting at $t = 0$ in state x_1, $u^* \rightarrow S - \sqrt{(h/c)}\,\psi x$ which means the optimal policy in such a case is to increase the production by an amount proportional to the deteriorated quantity, the proportionality constant being the square root of the ratio of holding cost to production cost.

Discounted Cost SPIM

When $T \rightarrow \infty$ even though u^*_{∞} have a plausible interpretation, we note that $V(t,x)$ tends to ∞. To keep $V(t,x)$ finite, we introduce a discounted factor ρ in which case it is easy to show that the OPR for a discounted cost SPIM is

$$(22)\quad u^*_{\rho} = S + \frac{\psi^2 Q}{c}\,x + \frac{1}{2}\left(\frac{B}{c} - 2S\right) e^{\rho(t-T)} \exp\left\{\frac{1}{c}\int_t^T \psi^2 Q\,ds\right\}$$
$$+ \frac{B_1}{2c}\int_t^T \phi(s)\exp\left\{\frac{1}{c}\int_t^s (\psi^2 Q - \rho c)\,du\right\} ds$$

$$\text{where } Q = \frac{c}{\psi^2}\,\frac{m_1(T)m_2 - m_2(T)m_1 y}{m_1(T) - m_2(T)y}, \quad y = \exp\left\{-\int_t^T (m_1 - m_2)\,ds\right\}$$

$$m_1, m_2 = \frac{1}{2}\left[\rho \pm \sqrt{\left(\rho^2 + 4h\psi^2/c\right)}\right] \text{ and } m_1(T), m_2(T) = \frac{1}{2}\left[\rho \pm \sqrt{\left(\rho^2 + 4h\psi^2(T)/c\right)}\right]$$

The asymptotic behaviour of u^*_{ρ} as $T \rightarrow \infty$ indicate that $u^*_{\rho(\infty)} \rightarrow S + m_2 x$ so that for a long process the change in production due to deterioration would be $\left[\sqrt{\left(\frac{\rho^2}{4} + \frac{h}{c}\right)} - \sqrt{\left(\frac{\rho^2}{4} + \frac{h}{c}\psi^2\right)}\right] x$ in order to meet a constant demand of S units per unit of time.

4. SPIM when Demand Rate is Varying

If we assume a deterministic demand rate then the OPR becomes

$$(23)\quad u^* = S + \sqrt{(h/c)}\,\frac{y-1}{y+1}\,\psi x + \frac{1}{2}\left(\frac{B}{c} - 2S(T)\right) \exp\left\{\frac{1}{c}\int_t^T \psi^2 Q\,ds\right\}$$
$$+ \int_t^T \frac{dS}{ds} \exp\left\{\frac{1}{c}\int_t^s \psi^2 Q\,du\right\} ds + \frac{B_1}{2c}\int_t^T \phi(x)\exp\left\{\frac{1}{c}\int_t^s \psi^2 Q\,du\right\} ds.$$

Comparing (23) with (14), we find that the adjustment needed in the OPR due to deterministic demand is

$$(24)\quad \Big[S - S(T)\Big]\exp\left\{\frac{1}{c}\int_t^s \psi^2 Q du\right\} + \int_t^T \frac{dS}{du}\exp\left\{\frac{1}{c}\int_t^s \psi^2 Q du\right\} ds$$

Thus we need just to obtain (24) and add it to (14) to obtain the OPR when demand is deterministic.

5. Concluding Remarks

The inclusion of the possibility of items in an inventory to deteriorate over time after some time from the entry into the inventory reflects reality in many applications. Further for various functional forms of deterioration rates and demand rates, our model yields OPRs. Finite and infinite horizon results for discounted as well as undiscounted cases yield results which intuition tells is correct.

The OPR are found to depend on the demand rate, the point of time from where deterioration starts, the on-hand inventory level, the deterioration rate and the distance from the planning horizon. Therefore in addition to remarks made in [3], we can say OPR would be positive for lower values of decay rate and inventory level. For high inventory level and high decay rate also, it is likely that OPR will be positive. Only high inventory level and low decay rate yield a negative production rate, and in such a case, disposal of items in the inventory should reduce the loss function through lowering of holding costs.

In this paper, attention was restricted to one-product inventory. Extensions to multi-product inventory can also be studied in similar lines. Besides the constant and deterministic demands, a probabilistic demand rate may be incorporated to make the model even more realistic. Bivariate controls on both the production rate and inventory level may also be a possible extension of this work.

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GLOSSARY OF MATHEMATICAL TERMS
(Proposed)
(T)

Table सारिणी
Tabular सारणीबद्ध
Tabulate सारणीमा भर्नु
Tabulator सारणीयंत्र, सारणीयक
Tabulation सारणीयन
Tangency स्पर्शिता
Tangent स्पर्शरेखा, स्पर्शी
Tangential स्पर्शीय, स्पर्शरेखीय
Tagentially स्पर्शीयत
Target लक्ष्य
Tau टाउ (τ)
Tautology पुनरावृत्ति
Telescopic अत: सर्पी, दूरदर्शक
Ten दश
Tenth दशौं
Tend प्रवृत्त हुनु
Tendency प्रवृत्ति
Tending प्रवृत्तित
Tenfold दशगुना
Tensile तनन्
Tension तनाव
Tensor टेन्सर, प्रदिश
Term पद
Terminus
Terminal टर्मिनल, अन्तिम, अंतस्थ
Terminate सांत, अन्त हुनु
Terminating सान्त
Termination समापन, अंत
Termivise पदश:
Tertairy तरशियरी, तृतीयक
Test परीक्षण
Tetrad चतुष्ट्य, चतुष्क
Tetragon चतुष्कोणीय
Tetrahedral चतुष्फलकीय
Tetrahedron चतुष्फलक
Theorem प्रमेय
Theoretical सैद्धांतिक
Theoretically सिद्धान्तत्:
Theory सिद्धान्त
Theta थेटा (Θ, θ)
Third तेश्रो
Thirteen तेह्र
Thirteenth तेह्रौं
Thrity तीस
Thirtieth तीसौं
Thousand हजार
Thousandth हजारौं
Thrice तेव्वर
Thrust प्रणोद, क्षेप
Time समय, अवधि, काल

यो शब्दावली त्रि.वि., कीर्तिपुर बहुमुखी क्याम्पस, गणित तथा नेपाली शिक्षण समितिले संयुक्त रूपमा तयार गरिएको हो ।

Topological संस्थितिक
Topologically संस्थितिकत:
Topology संस्थितिकी
Torque बल आघुर्ण, टर्क
Torsion मरोड, विमोटन
Torsional मरोड
Torus टोरस, वृत्तज ठोसवलय
Tossing संपरिक्षेपण
Total जोड, योग
Touch स्पर्श
Trace अनुरेख, ट्रेस
Tracing अनुरेखन
Trajectory प्रक्षेप्यपथ, प्रपथ
Transcendental अबीजीय
Transfer स्थानान्तर
Transform रूपान्तर
Transformation रूपान्तरण
Transit संक्रमण
Transition संक्रमण
Transitional संक्रामी
Transitive संक्रामक
Translation स्थानान्तरण
Transpose परिवर्त
Transposition पक्षांतरण
Transverse अनुप्रस्थ
Transversal तिर्यक छेदीरेखा, अनुप्रस्थत:
Transversally अनुप्रस्थत:
Trapezium समलंब, ट्रैपिजियम
Treble तेव्वर
Trend प्रवृत्ति, उपनति
Triad त्रय
Trial अभिप्रयोग
Triangle त्रिभुज, त्रिकोण
Triangular त्रिभुजीय, त्रिकोणीय
Triangulation त्रिभुजन, त्रिकोणायन
Tricuspid त्रि-अंकीय
Tricuspid त्रिवलन
Trigonometric त्रिकोणमितीय
Trigonometrical त्रिकोणमितीय
Trigonometrically त्रिकोणमितित:
Trigonometry त्रिकोणमिति
Trihedral त्रिफलक
Trilinear त्रिरेखीय
Triple त्रिगुण, तीनगुना, तेव्वर
Trivariant त्रिचर
Trivial तुच्छ, साधारण
True सत्य
Truth सत्यता, यथार्थता
Truncated लुनाग्र
Truncation छेदन, छिन्नकरण लुनाग्रता
Turbulence प्रक्षोभ
Turbulent प्रक्षुब्ध

Turning वर्तन

Twelve बाह्र

Twelfth बाह्रौं

Twenty बिस

Twentieth बिसौं

Twice दकल, दोव्वर

Two दुई

Typical प्रतिरूपी

(U)

Ultimate चरम, अन्तिम, परम

Ultrafilter अति सूक्ष्म निस्यंदक

Ultrafine परा सूक्ष्म

Ultraspherical परागोलीय

Umbilic शून्य वृत्तक, नाभि

Umbilical नाभिकीय

Unaccelerated अत्वरित

Unbounded अपरिवद्ध

Uncertainty अनिश्चितता

Unconditional अप्रतिबंधन

Unconstrained अप्रतिबंधित

Uncountable अगणनीय

Undetermined अनिर्धारित

Undistorted अविकृत

Unextended अविस्तारित

Ungrouped अवर्गीकृत

Uniaxial एक अक्षीय

Unicomponent एक घटकी, एक अवयवी

Unidirectional एक दिशीय

Uniform एक समान

Uniformity एक समानता

Uniformly एक समान रूपले

Unimodal एक बहुलकी

Unimodular एक मात्रांकी

Unique अद्वितीय, एकमात्र, अनन्य

Uniquely अविकल्पत: अद्वितीयत:

Union सम्मिलन, संयोग, युनियन

Uniplanar समतलिय

Uniqueness अद्वितीयता

Unit मात्रक, एकाई, यूनिट, तत्समक

Unitary ऐकिक

Unity एक

Univalent एकमानी

Univalence एकमानता

Univariant एकचर

Univariate एकविचर

Universal सार्वत्रिक

Universe विश्व, समष्टि

Unknown अज्ञात

Unlike विजातीय, विपरीत

Unlimited असीमित

Unnecessary अनावश्यक

Unordered अक्रमित

Unprimed अशिरवी

Unrestricted असीमित

Unsatisfactory अमान्य, असंतोषप्रद

Unstable अस्थायी, अस्थीर

Unsymmetrical असममित

Upthrust उप्लावन

Utility उपयोगिता, उपयोग

(V)

Vacuous रिक्त

Vacuum वैकुम, खाली, रिक्त स्थान

Valid वैध, मान्य

Validity वैधता, मान्यता

Valuation मानाकन

Value मूल्य, मान

Vanish लुप्त हुनु, शून्य हुनु, हराउनु

Variable चर, चल

Variance प्रसरण

Variant परिवर्त

Variate विचर

Variation विचरण

Variational विचरणी

Vary विचरित हुनु, परिवर्तित हुनु

Varying परिवर्ती

Vector वेक्टर, सदिश

Velocity वेग

Verifiable सत्यापनीय

Verify जांच्नु

Verification सत्यापन, जांच

Vertex शीर्ष

Vertical उध्वाधिर, उदग्र, ठाडो

Vibration कम्पन

Vinculum रेखाकोष्ठक

Virtual कल्पिट

Viscid विस्कासी

Viscous विस्कासी

Viscosity विस्कासीता

Void शून्य, शुन्यिका

Volume आयतन

Voluntary ऐच्छिक

Vortex वोर्टैक्स, भ्रमरी, भूमरी

Vorticity भूमरीय

(W)

Wave तरंग, छाल

Weak निर्बल

Weight तौल, भार

Weighted भारित, तौलित

Weigh तौल्नु, जोख्नु

Wedge वेज्, वट्

Wheel पांग्रा, चक्का

Wide चौडा

Width चौथाई

Work काम

(Y)

Yard गज

Year वर्ष

Yearly वार्षिक

Ypsilon यूपसिलन (Υ, υ)

(Z)

Zenith खमध्य, शिरोविन्दु

Zeta जिता (Ζ, ζ)

Zodiac राशिचक्र

Zone मंडल, जोन, क्षेत्र, प्रदेश