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Mathematical model to describe the distribution of female age at marriage

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Abstract: This paper has attempted to study the pattern of age at marriage by using a mathematical model. Model proposed by Mishra [1] has been applied and modified under some specified assumptions. The data are taken from a sample survey of Palpa and Rupandehi Districts. Other sources of data have also been utilized for testing the suitability of the model. The proposed model was found to be an appropriate model for describing the distribution of females according to age at marriage in the developing countries like Nepal and neighboring country India.

1. Introduction

Female age at marriage is an important demographic variable due to its influence on fertility especially in the developing countries with a low rate of contraceptive use. Marriage usually puts the foundation of family formation and, as such, is an important determinant of fertility associated with the duration of exposure for the risk of childbearing [2, 3]. Marriage in Nepal is universal and an early age at marriage is observed for both the males and females.

Several authors have used a number of probability models such as lognormal distribution[4], convolution of a normal and exponential distribution [5], linear function of the logarithm of a standard gamma distribution [6], two parameter log-logistic model, Gompertz curve, simple polynomials and logistic curve [7, 8, 9] convolution of two exponential distributions [10] type I extreme value distribution [11, 12, 13] to fit and graduate the distribution of females according to their age at marriage. However, these models are conceptually difficult to understand and computation of parameters as well these models provide a large discrepancy between observed and expected values. Likewise a number of models were used in order to describe the distribution demographic parameters [14, 15, 16, 17, 18]. However a very simple mathematical model was proposed by Mishra [1] to describe the data on the age at first marriage. This simple model is applied here with some modification to study the distribution of females according to their age at marriage. In brief, it is given in the following section.

2. The data

This study is based on the data taken from a sample survey entitled "Demographic Survey on Fertility and Mobility in Rural Nepal (DSFM): A Study of Palpa and Rupandehi Districts" conducted between January and June 2000 [1]. A total of 811 households were surveyed. The data of NFHS 1996 has been utilized. More over, the data from India, UP [13] and Assam [12] have also been utilized for testing the suitability of the model.

3. Model

Let x be a variable which takes the value 0, if the female is married before attaining 12 years of age, 1, if she is married between 12–15 years, 2, if between 15–18 years, 3, if 18–21 years, and so on. The variable x can also be regarded as a number of failures preceding the first success. Researchers have not assumed constant probability of success and independence of trials by discussing the distribution of marriage and first birth [19, 20]. The probability p_i of getting a success in the $(i+1)^{\text{th}}$ trial when it is known that first i trials resulted in failure, increases as $i(i = 0, 1, 2, 3, \ldots k)$ moves from zero to a certain value s and decreases monotonically as i moves from s to a value $t(\geq s)$ and thereafter remains constant. That is, the probability that the female is married in i^{th} age group (i.e. $x = i$, for $i = 0, 1, 2, 3, \ldots k$) then,

$$P(x=i) = P(x \geq i) * P(x=i/x \geq i) \tag{1}$$

In other words, the probability can be expressed as the product of the probabilities that the same did not marry in the preceding i age groups i.e. failure $(1-p_i)$ and the same marries in the i^{th} age-group given that she did not marry in the preceding i^{th} age-groups i.e. success (p_i). i.e.

(2) $P(x=i/x\geq i)=p_i$ and $P(x=i)=1-p_i=q_i$

and $P(x=0)=p_0$; as $P(x\geq 0)=1$,

$$P(x=1)=q_0\,p_1$$
$$P(x=2)=q_0\,p_1\,p_2$$
$$P(x=3)=q_0\,q_1\,q_2\,p_3$$
$$\cdots$$
$$\cdots$$
$$P(x=k)=q_0\,q_1\,q_2\,\cdots\quad\cdots\,q_{k-1}\,p_k$$
$$P(x\geq k)=q_0\,q_1\,q_2\,\cdots\quad\cdots\,q_{k-1}\,q_k$$

The different probabilities $p_0, p_1, p_2, \ldots$ are defined in the following ways,

$$p_0=a$$
$$p_1=a+b$$
$$p_2=a+rb \text{ and}$$

(3) $p_i=a+(r+c)b$ for all $i=3,4,5,\ldots k$.

where a,b,c and r are the four parameters of the model.

However, in some societies the probability $p_i (i=3,4,5,\ldots k)$ is found not constant due to the occurrence of some marriages at the late ages and it decreases as i increases. For this purpose a number of decreasing factors have been tried for getting the decreasing probability p_i after $i=4,5,\ldots,k$ and it was found that an appropriate decreasing factor may be $c/(i-3)$. Thus replacing c by $c/(i-3)$ in equation (3) we get

(4) $p_i=a+\{r+c/(i-3)\}b$,

i.e. start decreasing from $i=5$, where $i=4,5,\ldots,k$ and $i\neq 3$ and the respective different probabilities are:

(4a) $p_4=a+(r+c)\,b$

(4b) $p_5=a+(r+c/2)\,b$

(4c) $p_6=a+(r+c/3)\,b$

(4d) $p_7=a+(r+c/4)\,b$, and so on.

It is observed that the model (4) has four parameters and the expected frequencies would be exactly equal to the observed frequencies for $x=0,1,2$. The difference between the observed and expected frequencies would start from x=3 onwards. Thus, for a small value of k, the model (4) may not be appropriate. Model (4) is now modified as

(5) $p_i = a + (r + b)r$ for all $i = 3, 4, 5, \ldots k$.

This model has only three parameters, and these have been estimated by using iteration or the maximum likelihood method. For this, Mishra [1] has given a likelihood function as,

(6) $$L = a^{f_0}(1-a)^{N-f_0}(a+b)^{f_1}(1-a-b)^{N-f_0-f_1}(a+rb)^{f_2}(1-a-rb)^{N-f_0-f_1-f_2}$$
$$\{a+(r+c)b\}^{\Sigma f_i}\{1-a-(r+c)b\}^T$$

where $f_0, f_1, f_2, \ldots f_{k-1}, f_k$, are the respective frequencies for $i = 0, 1, 2, \ldots k-1, k$; $N = f_0 + f_1 + f_2 + \ldots + f_k$ and $T = f_4 + 3f_5$

Taking logarithms in (3) and solving, we get the following estimating equations:

(6a) $f_0 / a - (N - f_0)/(1-a) = 0$

(6b) $f_1 / (a+b) - (N - f_0 - f_1)/(1-a-b) = 0$

(6c) $f_2 / (a+rb) - (N - f_0 - f_1 - f_2)/(1-a-rb) = 0$

(6d) $\Sigma f_i / [a + (r+c)b] - T / [1 - a - (r+c)b] = 0$

Solving above equations one can estimate the parameters (a, b, c and r) of the models (3) and (4). Similar procedure has also been used to estimate the parameters (a, b and r) of the model (5).

4. Applications

The proposed model has been fitted to the data on age at marriage for females residing in *Hills*, *Tarai*, and rural Nepal (Tables 1 and 2). Data of NFHS 1996 as well as the data from UP, India [13] and Assam, India [12] have also been tested (Tables 2 and 3). The chi-square values suggest that the models (3), (4) and (5) fit well to all the data set on age at marriage. Chi-square values suggested that the proposed model was more powerful to describe the distribution of females according to the age at marriage in Nepal and India. Mean age at marriage was found 17.0 and 16.5 years for females residing in *Hills* and *Tarai* respectively whereas it was 16.7 years for females residing in rural Nepal. The mean age at marriage was 16.4 years for the NFHS 1996 data.

Table 1 Observed and expected distribution of female age at marriage.

Re-scale	Hill, DSFM, 2000				Tarai, DSFM, 2000			
age	Obs	Exp.			Obs.	Exp.		
		(3)	(4)	(5)		(3)	(4)	(5)
0	10	10.00	10.00	10.00	26	26.00	26.00	26.00
1	82	82.00	82.00	82.00	104	104.00	104.00	104.00
2	197	197.00	197.00	205.86	206	206.00	206.00	205.60
3	103	114.58	115.11	106.45	166	166.88	167.31	163.92
4	50	42.56	42.76	41.19	35	36.20	36.28	38.09
5	19	15.81	14.53	15.94	8	7.85	6.41	8.85
6	6	5.87	5.99	6.17	2	1.70	2.15	2.06
7	3	2.18	2.61	2.39	2	0.37	0.85	0.48
Total	470	470.00	470.00	470.00	549	549.00	549.00	549.00
χ^2.		3.22	3.87	2.29		0.484	0.765	0.131
d.f.		2	2	3		1	1	2
Parameter								
a		0.021277		0.021277		0.047359		0.047359
b		0.156984		0.156984		0.151494		0.151494
r		3.184320		3.293200		2.932711		2.925300
c		0.683560		-		1.923783		-

Table 2 Observed and expected distribution of female age at marriage.

Rescale age	Rural Nepal, DSFM, 2000				NEPAL, NFHS, 1996			
	Obs.	Exp.				Exp		
		(3)	(4)	(5)	Obs.	(3)	(4)	(5)
0	36	36.00	36.00	36.00	309	309.00	309.00	309.00
1	186	186.00	186.00	186.00	2189	2189.00	2189.00	2189.00
2	403	403.00	403.00	422.90	3741	3741.00	3741.00	3701.74
3	269	277.84	279.05	241.78	1549	1568.09	1568.09	1598.01
4	85	82.36	82.72	86.44	450	445.89	446.15	453.12
5	27	24.41	21.07	30.89	139	126.80	119.34	128.44
6	8	7.24	7.88	11.04	39	36.05	38.18	36.43
7	5	2.15	3.28	3.95	6	10.25	12.88	10.33
8	-	-	-	-	7	2.92	4.45	2.93
Total	1019	1019.0	1019.0	1019.00	8429	8429.00	84296.00	8429.00
χ^2		4.52	2.98	5.63		9.22	8.87	9.89
d.f.		3	3	4		4	4	5
Parameter								
a		0.035329		0.035329		0.036666		0.03666
b		0.153888		0.153888		0.232922		0.23292
r		3.056233		3.209100		2.550610		2.52165
c		1.286165		-		0.36450		-

Table 3: Observed and expected distribution of female age at marriage

Rescale	INDIA, UP (Sinha,1998)				India, Assam (Nath & Talukdar, 1992)			
Age		Exp.				Exp.		
	Obs.	(3)	(4)	(5)	Obs.	(3)	(4)	(5)
0	247	247.00	247.00	247.00	221	221.00	221.00	221.00
1	564	564.00	564.00	564.00	694	694.00	694.00	694.00
2	962	962.00	962.00	960.86	240	240.00	240.00	235.28
3	776	772.93	772.95	768.85	105	102.57	102.81	111.03
4	182	195.07	194.02	196.74	21	25.50	25.56	19.83
5	56	47.43	44.79	50.37	8 }	6.34	5.88	7.17
6	15 }	12.09	14.23	12.88	2 }	1.58	1.75	2.69
7	2 }	3.48	5.01	3.30	-	-	-	-
Total	2804	2804.00	2804.00	2804.00	1291	1291.0	1291.00	1291.00
$\chi2$		2.93	3.19	1.84		1.54	1.49	0.69
d.f.		2	2	3		1	1	2
Parameter								
a		0.088090		0.088090		0.171185		0.17118
b		0.132483		0.132483		0.477413		0.47741
r		2.978513		2.97420		0.978425		0.94455
c		1.991741		-		0.236867		-

5. Conclusions

The model proposed for describing the distribution of females according to age at marriage in Nepal was found an appropriate distribution. The proposed model is also fit well to the data of India. Hence the proposed model may be used to describe the distribution of females according to age at marriage in the developing countries like Nepal, India, Bangladesh, etc. The model also provided the average age at marriage of about 18 and 17 years for females residing in *Hills* and *Tarai* respectively and 17 years for females residing in rural Nepal, which was found very close with the median age at marriage while computed using all females (married as well as unmarried females) [2].

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